
General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Write complete sentences, and use notation correctly. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate. Eschew obfuscation.*

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1. (80 pts.) Solve each of the following differential equations or initial value problems. If there is no initial condition, obtain the general solution. [20 points/part]

(a) $\frac{dy}{dx} = \tan^2(x)\sec(y)$; $y(0) = \frac{\pi}{6}$.

(b) $x\frac{dy}{dx} + y = (xy)^{3/2}$ for $x > 0$.

(c) $\frac{dr}{d\theta} + 2\tan(\theta)r = 4\cos^3(\theta)$ *and* $r(0) = 4$

(d) $(x+2y) + (2x+y)\frac{dy}{dx} = 0$

2. (6 pts.) What conditions on the coefficients of the following homogeneous equation are sufficient for the equation to be exact?

$$(Ax^2 + Bxy + Cy^2)dx + (Dx^2 + Exy + Fy^2)dy = 0$$

Do not attempt to solve the DE.

3. (6 pts.) Every solution to the differential equation $y'' - 16y = 0$ is of the form $y(x) = c_1 e^{4x} + c_2 e^{-4x}$. Which of these functions satisfies the initial conditions $y(0) = 4$ and $y'(0) = 8$??

4. (8 points) The following differential equation may be solved by either performing a substitution to reduce it to a separable equation or by performing a different substitution to reduce it to a homogeneous equation. Display the substitution to use and perform the reduction, **but do not attempt to solve the separable or homogeneous equation you obtain**.

$$(x - 2y + 2)dx + (4x - 8y - 6)dy = 0$$

10 Point Bonus: Although you will later learn a body of theory that will make the solution of the following following 2rd order equation routine, the truth is that you can actually obtain the general solution to the ODE with the knowledge you have now.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x$$

Show how in detail. [Say where your work is below! There isn't room here.]