General directions: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Eschew Obfuscation. Show me all the magic on the page.*

1. (30 pts.) Obtain the general solution to each of the following linear homogeneous constant coefficient equations.

(a) $y''(x) - 7y'(x) + 10y(x) = 0$

(b) $y''(x) - 4y'(x) + 5y(x) = 0$

(c) $\dfrac{d^5y}{dx^5} + 4\dfrac{d^3y}{dx^3} = 0$

2. (10 pts.) Find the unique solution to the initial value problem

$$y'' - y' = 2e^x \quad ; \quad y(0) = 2, \quad y'(0) = -2,$$

given that a fundamental set of solutions to the corresponding homogeneous equation is { 1, e^x } and a particular integral of the original ODE is

$$y_p(x) = 2xe^x.$$

Hint: Save time. Use the stuff served on the platter with the cherry on top.

3. (15 pts.)   Find a particular integral, y_p, of the differential equation

$$y'' + 4y = 4\csc(2x).$$

Do not write the general solution.

4.   (10 pts.) Set up the correct linear combination of undetermined coefficient functions you would use to find a particular integral, y_p, of the O.D.E.

$$y'' - 6y' + 10y = 4\sin(x) + 5xe^{3x} - 10\cos(x)e^{3x}.$$

[**Warning:** *(a) If you skip a critical initial step, you will get no credit!! (b) Do not waste time attempting to find the numerical values of the coefficients!!*]

5.   (10 pts.) The factored auxiliary equation of a certain homogeneous linear O.D.E. with real constant coefficients is as follows:

$$(m - 1)^4(m - (\pi + i))^2(m + (\pi - i))^2 = 0$$

Note: The plus sign in the factor $(m + (\pi - i))^2$ above is a typographical error.

(a) (5 pts.) Write down the general solution to the differential equation. [WARNING: Be very careful. This will be graded Right or Wrong!!]

(b) (5 pt.) What is the order of the differential equation?

6. (15 pts.)   (a) Obtain the differential equation satisfied by the family of curves defined by the equation (*) below.

(b) Next, write down the differential equation that the orthogonal trajectories to the family of curves defined by (*) satisfy.

(c) Finally, solve the differential equation of part (b) to obtain the equation(s) defining the orthogonal trajectories. [These, after all, are another family of curves.]

$$(*) \quad y = x - 1 + ce^{-x} .$$

7.   (10 pts.)   The nonzero function $f(x) = \exp(x)$ is a solution to the homogeneous linear O.D.E.

$$(*) \qquad (x-1)y'' - xy' + y = 0$$

(a) Reduction of order with this solution involves making the substitution

$$y = ve^x$$

into equation (*) and then letting $w = v'$. Do this substitution and obtain in standard form the first order linear homogeneous equation that w must satisfy. (b) Finally, obtain an integrating factor, μ, for the first order linear ODE that w satisfies and then stop. ***Do not attempt to actually find v.***

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Silly 10 Point Bonus: Suppose that the function

$$f(x) = x^n ,$$

where n is a positive integer, is a solution to the constant coefficient homogeneous linear equation

$(*) \qquad a_m y^{(m)} + a_{m-1} y^{(m-1)} + \ldots + a_2 y'' + a_1 y' + a_0 y = 0$

where $m > n$. What can you say about the coefficients of the ODE, and what can you say about its fundamental set of solutions?? Why???
[Say where your work is, for it won't fit here.]