NAME: Brief Answers by O.O. TEST1A/MAP2302 Bonus

Silly 10 Point Bonus: Frodo asked Gandalf, "Do you know of a closed form for the power series function

$$f(x) = \sum_{k=1}^{\infty} \frac{1}{k2^k}(x-4)^k \quad ?$$

I know the function is defined on the interval I = [2,6)." Gandalf stood silent for a few minutes with a furrowed brow and then replied, "Of course. The closed form is an alias for the function f, which is the solution to an initial value problem to which the magical Fundamental Theorem may be applied." Then Gandalf vanished mysteriously after leaving behind a rapidly fading cheshire cat grin.

Help Frodo.

(a) What is the easy to solve IVP ?? (b) Reveal to Frodo the other identity of the function f. //

(a) First, it is obvious that f(4) = 0. Then, by differentiating the power series function f term-wise or term-by-term, we obtain

$$\begin{aligned} f'(x) &= \sum_{k=1}^{\infty} \frac{k}{k2^k}(x-4)^{k-1} \\ &= \sum_{k=1}^{\infty} \frac{1}{2^k}(x-4)^{k-1} \\ &= \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)\left(\frac{x-4}{2}\right)^{k-1} \\ &= \sum_{j=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{x-4}{2}\right)^{j} \\ &= \frac{\frac{1}{2}}{1-\left(\frac{x-4}{2}\right)} = \frac{1}{6-x} \end{aligned}$$

for $|x - 4| < 2$ since f′ is really an easy-to-sum geometric series in disguise. Consequently, the IVP is simply $f'(x) = (6-x)^{-1}$ and $f(4) = 0$ for $|x - 4| < 2$.

(b) Finally, the magical Fundamental Theorem of Calculus provides us with the other name for f, thus:

$$f(x) = \int_4^x \frac{1}{6-t}\,dt = \ln(2) - \ln(6-x) = \ln\left(\frac{2}{6-x}\right)$$

for $|x - 4| < 2$.