General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Be very careful. What is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate.*

1. (85 pts.) Solve each of the following differential equations or initial value problems. Show all essential work neatly and correctly. [17 points/part]

(a) $(9y^2 + 1)dx + (9 \cdot \sec(x))dy = 0$

(b) $3y' + x^{-1}y = 6xy^{-2}$ with $y(1) = -2$.

(c) $(9x^2 + y^2)dx + (x^2 - xy)dy = 0$

(d) $(ye^{xy} + 8x^3)dx + (xe^{xy} - 6y^2)dy = 0$

(e)

$$y' + y = f(x) \text{ , where } f(x) = \begin{cases} 4 & \text{, for } 0 \le x < 2 \\ 2x & \text{, for } 2 \le x \end{cases}$$

and $y(0) = -1$.

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2. (5 pts.)   It is known that every solution to the differential equation $y'' + 4y = 0$ is of the form

$$y = c_1 \cdot \sin(2x) + c_2 \cdot \cos(2x).$$

Which of these functions satisfies the initial conditions $y(\pi/4) = 2$ and $y'(\pi/4) = -4$?? [Hint: Determine c_1 and c_2 by solving an appropriate linear system. Don't waste time verifying y, above, is a solution.]

3. (10 pts.)   (a) It is known that $f(x) = x^r$ is a solution to the ordinary differential equation

$$(*) \quad x^2y'' + 3xy' - 8y = 0$$

for certain values of the constant r. Determine all such values of r.

(b) If the differential equation

$$(**) \quad (Rx + Sy)dx + (Tx + Uy)dy = 0$$

is exact, what must be true about the constants R, S, T, and U?

Silly 10 Point Bonus: Frodo asked Gandalf, "Do you know of a closed form for the power series function

$$f(x) = \sum_{k=1}^{\infty} \frac{1}{k3^k}(x - 5)^k \ ?$$

I know the function is defined on the interval I = [2,8)." Gandalf stood silent for a few minutes with a furrowed brow and then replied, "Of course. The closed form is an alias for the function f, which is the solution to an initial value problem to which the magical Fundamental Theorem may be applied." Then Gandalf vanished mysteriously after leaving behind a rapidly fading cheshire cat grin.
     Help Frodo.
(a) What is the easy to solve IVP ?? (b) Reveal to Frodo the other identity of the function f. // Say where your work is here: