Read me first: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Be very careful. Remember that what is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate.*

1. (40 pts.) Solve each of the following second order differential equations or initial value problems. Be very careful. Show all essential work. Do not write nonsense.

(a) $y'' + 10y' + 21y = 0$

(b) $d^3y/dx^3 + 25(dy/dx) = 0$

(c) $y'' - 12y' + 36y = 0$

(d) $y'' - 2y' = 10$; $y(0) = -2$, $y'(0) = 1$

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2. (5 pts.) The factored auxiliary equation of a certain homogeneous linear O.D.E. with real constant coefficients is as follows:

$$m^2(m - \pi)^2(m - (1+2i))^3(m - (1-2i))^3 = 0$$

(a) (3 pts.) Write down the general solution to the differential equation. [**WARNING: Be very careful. This will be graded Right or Wrong!!**] (b) (2 pt.) What is the order of the differential equation?

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3. (10 pts.) Given that $f(x) = \sin(3x)$ is a solution of the homogeneous linear O.D.E. $y'' + 9y = 0$, use only the method of reduction of order to find a second, linearly independent solution. [**WARNING: No reduction, no credit!! Show all the steps of this neatly while using notation correctly. The integral formula for the second solution does not suffice.**]

4. (20 pts.) **Using the method of variation of parameters, not the method of undetermined coefficients**, find a particular integral, y_p, for the differential equation

$$y'' - 2y' = 10.$$

[Hint: Read this problem twice and do exactly what is asked to avoid heartbreak!! *Do not obtain* y_p *using the method of undetermined coefficients*.]

5. (5 pts.) Consider the linear homogeneous ODE

(**) $$y''(x) + \cos(x)y'(x) + x^2y(x) = 0.$$

Suppose $f_1(x)$, $f_2(x)$, and $f_3(x)$ are three solutions to (**) defined on the real line with

$$f_1(1) = 1 \text{ and } f_1'(1) = -2 ;$$

$$f_2(1) = -3 \text{ and } f_2'(1) = 0 \text{ ; and}$$

$$f_3(1) = -5 \text{ and } f_3'(1) = 10.$$

Let A = { f_1, f_2 } , B = { f_1, f_3 } , and C = { f_2, f_3 }. Which of the above sets is a fundamental set of solutions and which is not?? Explain how we know the difference. [Hint: You can't go wronskian by guessing.]

6. (10 pts.) Set up the correct linear combination of undetermined coefficient functions you would use to find a particular solution, y_p, for the O.D.E.

$$y'' - 6y' = 10x^2e^{6x} - 7\sin(2x) - 32x.$$

Do not attempt to actually find the numerical values of the coefficients!!

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7. (10 pts.) (a) Obtain the differential equation and initial condition needed to solve the following word problem. State what your variables represent using complete sentences. (b) Next, solve the initial value problem. (c) Then, answer the last part of the question. This will probably involve a second equation relating dependent and independent variables. [For (c), the exact value in terms of natural logs will suffice.]

//A large water tank initially contains 100 gallons of brine in which 40 pounds of salt is dissolved. Starting at time t = 0 minutes, a brine solution containing 2 pounds of salt per gallon flows into the tank at the rate of 5 gallons per minute. The mixture is kept uniform by a mixer which stirs it continuously, and the well-stirred mixture flows out at the same rate. When will the tank have a mixture containing 50 pounds of salt????//

Silly 10 Point Bonus: Evidently, $\{ 1, e^x \}$ is a fundamental set of solutions for the homogeneous linear constant coefficient ODE $y'' - y' = 0$. Is there a linear second order ODE with constant coefficients with $\{ x, 3e^x \}$ as a fundamental set of solutions? Proof?? Say where your work is here: