NAME: Ogre Ogre **TEST4/MAP2302** **Bonus Business**

Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Clearly show me the all the magic on the page.* Test #:

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10 Point Bonus: What's the exact value of the following definite integral: $\int_0^\infty |\sin(t)|e^{-t}\,dt$ Where's your work??

Hints:
- (a) Yes, this may be viewed in terms of the Laplace transform.
- (b) Yes, those are absolute value bars.
- (c) Yes, $|\sin(t)|$ is π-periodic.
- (d) No, you don't have to do integration by parts, but *tanstaafl* applies. Do you grok the unit stepping dance, the heavy side of thimble rigging??

First,

$$\int_0^\infty |\sin(t)|e^{-t}\,dt = \mathfrak{L}\{|\sin(t)|\}(1).$$

Since $|\sin(t)|$ is π-periodic and sine is non-negative on the interval $[0,\pi]$,

$$\mathfrak{L}\{|\sin(t)|\}(s) = \frac{\int_0^\pi \sin(t)e^{-st}\,dt}{1 - e^{-\pi s}}.$$

Now we may evaluate the definite integral above on the right side by viewing it as the Laplace transform of the function g defined by

$$g(t) = \begin{cases} \sin(t) & , \text{ for } 0 \le t \le \pi \\ 0 & , \text{ for } \pi < t. \end{cases}$$

Due to the observation that, except at $t = 0$ and $t = \pi$, we have $g(t) = \sin(t) - \sin(t)\cdot u_\pi(t)$, it follows that

$$\begin{aligned}\int_0^\pi \sin(t)e^{-st}\,dt &= \mathfrak{L}\{g(t)\}(s) \\ &= \mathfrak{L}\{\sin(t)\}(s) - \mathfrak{L}\{\sin(t)u_\pi(t)\}(s) \\ &= \frac{1}{1+s^2} - \mathfrak{L}\{h(t-\pi)u_\pi(t)\}(s), \text{ where } h(t-\pi) = \sin(t) \\ &= \frac{1}{1+s^2} - e^{-\pi s}\mathfrak{L}\{h(t)\}(s), \text{ where } h(t) = \ldots = -\sin(t) \\ &= \frac{1}{1+s^2} + \frac{e^{-\pi s}}{1+s^2}.\end{aligned}$$

Thus,

$$\int_0^\infty |\sin(t)|e^{-t}\,dt = \frac{1}{2}\frac{1 + e^{-\pi}}{1 - e^{-\pi}} = \frac{1}{2}\frac{e^{\pi} + 1}{e^{\pi} - 1}.$$