Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Clearly show me the all the magic on the page.* Test #:

1. (40 pts.) Without evaluating any integrals and using only the table provided, properties of the Laplace transform, and appropriate function identities, obtain the Laplace transform of each of the functions that follows:

(a) $h(t) = \begin{cases} -5 & , 0 < t < 10 \\ 22 & , 10 < t < 45 \\ -3 & , 45 < t \ . \end{cases}$

$\mathfrak{L}\{h(t)\}(s) =$

(b) $f(t) = \begin{cases} 8t & , 0 < t < 4 \\ 16 & , 4 < t \end{cases}$

$\mathfrak{L}\{f(t)\}(s) =$

(c) $(f*g)(t)$, when $f(t) = 2 \cdot e^{8t}\sin(3t)$ and $g(t) = 2t^4$

$\mathfrak{L}\{(f*g)(t)\}(s) =$

1.
(d) $g(t) = t^2 \cdot e^t \cdot \sin(t)$

Hint: Perform a deft prestidigitation with your magic writing wand correctly and one derivative is all that’s needed.

$\mathfrak{L}\{g(t)\}(s) =$

2. (10 pts.) (a) The Laplace transform of the following periodic function may be written in terms of a definite integral. Simply express the transform in terms of the appropriate definite integral, but do not attempt to evaluate that definite integral.

$$f(t) = \begin{cases} t^2 & \text{, for } 0 \le t < 2 \\ (t-4)^2 & \text{, for } 2 \le t < 4, \end{cases}$$

and $f(t) = f(t + 4)$ for $t \ge 0$.

$\mathfrak{L}\{f(t)\}(s) =$

(b) The following sum of definite integrals can be realized as the Laplace transform of a certain function g(t) defined for $t \ge 0$. Provide the precise definition of that function g.

$$\int_0^2 t^2 e^{-st} dt + \int_2^4 (t-4)^2 e^{-st} dt = \mathfrak{L}\{g(t)\}(s), \text{ where}$$

$g(t) =$

Warning: *Do not attempt to evaluate the Laplace transform of g.*

3. (15 pts.) Suppose that the Laplace transform of the solution to a certain initial value problem involving a linear differential equation with constant coefficients is given by

$$\mathcal{L}\{y(t)\}(s) = \frac{10e^{-\pi s}}{s^2 + 9} + \frac{8s + 49}{(s + 6)^2 + 9} .$$

What's the solution, y(t) , to the IVP??

y(t) =

4. (15 pts.) Using only the Laplace transform machine, very carefully solve the following very dinky first order initial value problem:

$$y' = f(t) \text{ , where } f(t) = \begin{cases} 4 & \text{, for } 0 \le t < 2 \\ 2t & \text{, for } 2 \le t \end{cases}$$

and y(0) = -1.

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5. (10 pts.) Using only the definition of the convolution in terms of a definite integral, not some shenanigans involving the Laplace transform, compute $(f*g)(t)$ when $f(t) = t$ and $g(t) = e^t$. Begin at the equal sign below.

$(f*g)(t) =$

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6. (10 pts.) Very neatly transform the given initial value problem into a linear system in $\mathcal{L}\{x\}$ and $\mathcal{L}\{y\}$ and stop. Do not attempt to solve for $\mathcal{L}\{x\}$ or $\mathcal{L}\{y\}$.

I.V.P.: $2x'(t) + y(t) = 12\cdot\delta(t - 8)$

$x(t) - 3y'(t) = 8\cdot e^{8t}$, $x(0) = -4$, $y(0) = 1$

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10 Point Bonus: What's the exact value of the following definite integral: $\int_0^\infty |\sin(t)|e^{-t}\,dt$ Where's your work??

Hints:
(a) Yes, this may be viewed in terms of the Laplace transform.
(b) Yes, those are absolute value bars.
(c) Yes, $|\sin(t)|$ is π-periodic.
(d) No, you don't have to do integration by parts, but *tanstaafl* applies. Do you grok the unit stepping dance, the heavy side of thimble rigging??