
General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. What is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate. Eschew obfuscation.*

Bonkers 10 Point Bonus: In an example where he solved a certain homogenous equation, Ross apparently used a table of integrals to evaluate the following integral:

$$\int \frac{1}{(v^2 + 1)^{1/2}}\, dv$$

On the back of Page 3 of 4, with your bare hands reveal all the magic in evaluating this integral. Happy Halloween.

Trig or treat! Set

$$v = \tan(\theta)$$

and restrict θ to the interval $(-\pi/2, \pi/2)$ so that

$$dv = \sec^2(\theta)d\theta$$

and

$$\sec(\theta) = (v^2 + 1)^{1/2}$$

as is usual for tangent substitutions.
Substituting, we have

$$\begin{aligned}\int \frac{1}{(v^2 + 1)^{1/2}}\, dv &= \int \frac{\sec^2(\theta)}{(\tan^2(\theta) + 1)^{1/2}}\, d\theta \\ &= \int \frac{\sec^2(\theta)}{(\sec^2(\theta))^{1/2}}\, d\theta \\ &= \int \sec(\theta)\, d\theta \\ &= \ln|\sec(\theta) + \tan(\theta)| + C.\end{aligned}$$

Replacing tangent and secant appropriately yields

$$\int \frac{1}{(v^2 + 1)^{1/2}}\, dv = \ln|(v^2 + 1)^{1/2} + v| + C.$$

Why didn't Ross simply assert that the integral could be obtained by a routine computation instead of referring to a table?? Hmmmm.