
General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. What is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate. Eschew obfuscation.*

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1. (75 pts.) Solve each of the following differential equations or initial value problems. If there is no initial condition, obtain the general solution. [15 points/part]

(a) $\sec(2y)\,dx + 20(x^2+1)\,dy = 0 \;;\; y(1) = 2\pi$

(b) $(y^2 - 4e^{2x})\,dx + (2xy + \ln(y))\,dy = 0$

(c) $(y^2 + xy + x^2)dx - (x^2)dy = 0$

(d) $5\frac{dy}{dx} + \frac{y}{x} = 20x^3y^{-4}$ *with* $x > 0$.

(e) $\frac{dr}{d\theta} + \tan(\theta) r = 2\theta\cos(\theta) \;;\; r(0) = 10$

2. (10 points) The following differential equation may be solved by either performing a substitution to reduce it to a separable equation or by performing a different substitution to reduce it to a homogeneous equation. Display the substitution to use and perform the reduction, **but do not attempt to solve the separable or homogeneous equation you obtain**.

$$(x - 2y - 3)dx + (2x + y - 1)dy = 0$$

3. (15 pts.) (a) Obtain the differential equation and initial condition(s) needed to solve the following word problem. State what your variables represent using complete sentences. (b) Next, solve the initial value problem. (c) Then, answer the last part of the question. [For (c), providing the exact value in terms of a suitable transcendental function will suffice, unless an algebraic function will do the job, in which case use the algebraic function.]

// Assume Newton's Law of Cooling:

A body with temperature of 100 °F is placed at time t = 0 in a medium maintained at a temperature of 20 °F. If, at the end of 10 minutes the temperature of the body is 60 °F, when will the body be 40 °F??

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Bonkers 10 Point Bonus: In an example where he solved a certain homogenous equation, Ross apparently used a table of integrals to evaluate the following integral:

$$\int \frac{1}{(v^2 + 1)^{1/2}}\, dv$$

On the back of Page 3 of 4, with your bare hands reveal all the magic in evaluating this integral. Happy Halloween.