**General directions:** *Show all essential work very neatly.   Use correct notation when presenting your computations and arguments. Write using complete sentences.  Be careful.    Remember this: "=" denotes "equals" ,  "⟹" denotes "implies" , and "⟺" denotes "is equivalent to".    Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate.  Show me all the magic on the page.*

1.   (15 pts.) Write down the general solution to each of the following linear constant coefficient homogeneous equations.

(a)    $y''(x) + 2y'(x) - 8y(x) = 0$

(b)    $y''(x) - 6y'(x) + 9y(x) = 0$

(c)    $\dfrac{d^4y}{dx^4} + 9\dfrac{d^2y}{dx^2} = 0$

2. (10 pts.)   The factored auxiliary equation of a certain homogeneous linear O.D.E. with real constant coefficients is as follows:

$$m(m - \pi)^2(m - (1+i))^2(m - (1-i))^2 = 0$$

(a)  (5 pts.) Write down the general solution to the differential equation. [**WARNING: Be very careful.  This will be graded Right or Wrong!!**] (b) (5 pt.) What is the order of the differential equation?

3. (10 pts.)    It turns out that the nonzero function
f(x) = e$^{2x}$ is a solution to the homogeneous linear O.D.E.

$$y'' - 4y = 0.$$

Using only the method of reduction of order, obtain a second,
linearly independent solution to this equation.
[**WARNING: No reduction, no credit!! Show all steps of this neatly
while using notation correctly. You are being graded on the
journey, not the destination.**]

4.    (15 pts.) Very carefully obtain the general solution to the
following Euler-Cauchy O.D.E. for x > 0.

$$x^2 y''(x) + xy'(x) - y = x^4.$$

5.    (10 pts.) Set up the correct linear combination of undetermined coefficient functions you would use to find a particular integral, $y_p$, of the O.D.E.

$$y'' + y = 4x - 2x\sin(x) + 7xe^x.$$

[**Warning:** *(a) If you skip a critical initial step, you will get no credit!!  (b)  Do not waste time attempting to find the numerical values of the coefficients!!*]

6. (15 pts.)    **Using the method of variation of parameters, not the method of undetermined coefficients**, find a particular integral, $y_p$, of the differential equation

$$y'' + y' = 4x$$

[**Hint: Read this problem twice and do exactly what is asked to avoid heartbreak!!**  *Do not obtain* $y_p$ *using the method of undetermined coefficients. Do not waste time getting the general solution.*]

7. (15 pts.)    Suppose

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

is a solution of the homogeneous second order linear equation

$$y'' - 2xy' = 0.$$

(a)  Obtain the recurrence formula for the coefficients of $y(x)$.
(b)  Which of the coefficients must be zero, no matter what the initial conditions may be?
(c)  If $y(x)$ also satisfies the initial conditions $y(0) = 0$ and $y'(0) = 1$, what is the numerical value of $c_5$ ??

8. (10 pts.)    Obtain the unique solution to the following initial value problem:

$$y''(x) - y'(x) = 4$$

$$with$$

$$y(0) = 2 \; and \; y'(0) = -1.$$

**Silly 10 Point Bonus:** Obtain a linear second order ODE with a fundamental set of solutions given by $\{ e^x , 1/x \}$   Say where your work is!