General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. What is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate. Eschew obfuscation.*

1. (90 pts.) Solve each of the following differential equations or initial value problems. Show all essential work neatly and correctly. [15 points/part]

(a) $\frac{dy}{dx} + \frac{3}{x}y = \frac{48}{x^2}$; $y(1) = 26$

(b) $\left(y(xy)^3 - 4e^{2x}\right)dx + \left(x(xy)^3 + 3e^{3y}\right)dy = 0$

(c) $\left(y^2 + xy - x^2\right)dx - \left(x^2\right)dy = 0$

(d) $2\cos^2(2y)dx + 10\sec(x)dy = 0$

(e) $7\frac{dy}{dx} + \frac{1}{x}y = \frac{24x}{y^6}$ for $x > 0$.

(f) $y'(x) + y(x) = f(x)$, where

$$f(x) = \begin{cases} x\ , & \text{if } 0 \le x < 3 \\ 3\ , & \text{if } 3 \le x. \end{cases}$$

and $y(0) = 0$

2. (5 pts.) It is known that every solution to the differential equation $y'' + y = 0$ is of the form

$$y(x) = c_1\sin(x) + c_2\cos(x).$$

Which of these functions satisfies the initial conditions $y(\pi/2) = 2$ and $y'(\pi/2) = 8$?? [Hint: Determine c_1 and c_2 by solving an appropriate linear system.]

3. (5 pts.) For certain values of the constant m the function $f(x) = x^m$ is a solution to the differential equation

$$x^2y''(x) - 6y(x) = 0.$$

Determine all such values of m.

Bonkers 10 Point Bonus: (a) The Fundamental Theorem of Calculus provides a neat formal solution involving a definite integral with respect to the variable 't' to the following dinky IVP:

$$y'(x) = \sin(x^2) \text{ and } y(0) = 1.$$

What is that solution? (b) Unfortunately $g(x) = \sin(x^2)$ cannot be integrated in elementary terms. Use the answer to (a), the Maclaurin series for sin(x), and term-wise integration, to obtain a power series solution to the IVP *using sigma notation*. [Say where your work is! You don't have room here!]