General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Write complete sentences, and use notation correctly. What is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate. Eschew obfuscation.*

1. (80 pts.) Solve each of the following differential equations or initial value problems. If there is no initial condition, obtain the general solution. [20 points/part]

(a) $\dfrac{dy}{dx} = 5x^4\cos^2(y)$; $y(0) = \dfrac{\pi}{3}$.

(b) $\left(4e^{2x} + y^2\right)dx + \left(3y^2 + 2xy\right)dy = 0$

(c) $\dfrac{dy}{dx} + \dfrac{1}{x}y = \dfrac{2}{x^2+1}$; $y(1) = \ln(4)$.

(d) $\left(x \sec\left(\dfrac{y}{x}\right) + y \right)dx - (x)\,dy = 0$; $y(1) = \dfrac{\pi}{6}$

2. (5 pts.) For certain values of the constant m the function $f(x) = x^m$ is a solution to the differential equation

$$x^2 y''(x) - 2x y'(x) = 0 \ .$$

Determine all such values of m.

3. (5 pts.) It is known that every solution to the differential equation $y'' - y = 0$ is of the form

$$y(x) = c_1 e^x + c_2 e^{-x} \ .$$

Which of these functions satisfies the initial conditions $y(0) = 2$ and $y'(0) = 8$??

4. (10 points) The following differential equation may be solved by either performing a substitution to reduce it to a separable equation or by performing a different substitution to reduce it to a homogeneous equation. Display the substitution to use and perform the reduction, **but do not attempt to solve the separable or homogeneous equation you obtain**.

$$(5x + 2y + 1)dx + (2x + y + 1)dy = 0$$

Bonkers 10 Point Bonus: (a) The Fundamental Theorem of Calculus provides a neat formal solution involving a definite integral with respect to the variable t to the following dinky IVP:

$$y'(x) = e^{x^2} \ \textit{and} \ \ y(0) = 1.$$

What is that solution? (b) Unfortunately the function

$$g(x) = e^{x^2}$$

cannot be integrated in elementary terms. Use the answer to (a), the Maclaurin series for e^x, and term-by-term integration, to obtain a power series solution to the IVP. Write your answer using sigma notation. [Say where your work is! You don't have room here!]