General directions: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (30 pts.) Obtain the general solution to each of the following linear homogeneous constant coefficient equations.

(a) $y''(x) - 3y'(x) - 10y(x) = 0$

(b) $y''(x) - 6y'(x) + 10y(x) = 0$

(c) $\dfrac{d^5y}{dx^5} + 5\dfrac{d^4y}{dx^4} = 0$

2. (10 pts.) Find the unique solution to the initial value problem

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2e^x \; ; \quad y(0) = 1, \;\; y'(0) = -1,$$

given that a fundamental set of solutions to the corresponding homogeneous equation is { 1, e^x } and a particular integral to the original ODE is

$$y_p(x) = 2xe^x.$$

Hint: Save time. Use the stuff served on the platter with the cherry on top.

3. (15 pts.) Find a particular integral, y_p, of the differential equation

$$y'' + y = 4\sec(x).$$

4. (10 pts.) Set up the correct linear combination of undetermined coefficient functions you would use to find a particular integral, y_p, of the O.D.E.

$$y'' - 4y' + 5y = x^2e^{-2x} + \sin(x)e^{2x}.$$

[**Warning:** *(a) If you skip a critical initial step, you will get no credit!! (b) Do not waste time attempting to find the numerical values of the coefficients!!*]

5. (10 pts.) The factored auxiliary equation of a certain homogeneous linear O.D.E. with real constant coefficients is as follows:

$$m(m - 2\pi)^2(m - (5i))^3(m - (-5i))^3 = 0$$

(a) (5 pts.) Write down the general solution to the differential equation.
[**WARNING: Be very careful. This will be graded Right or Wrong!!**]
(b) (5 pt.) What is the order of the differential equation?

6. (15 pts.) (a) Obtain the differential equation satisfied by the family of curves defined by the equation (*) below.
(b) Next, write down the differential equation that the orthogonal trajectories to the family of curves defined by (*) satisfy.
(c) Finally, solve the differential equation of part (b) to obtain the equation(s) defining the orthogonal trajectories. [These, after all, are another family of curves.]

(*) $x^2 = 2y - 1 + ce^{-2y}$.

7. (10 pts.) It turns out that the nonzero function $f(x) = \exp(x)$ is a solution to the homogeneous linear O.D.E.

$$(*) \qquad y''' - y = 0.$$

(a) Reduction of order with this solution involves making the substitution

$$y = ve^x$$

into equation (*) and then letting $w = v'$. Do this substitution and obtain the constant coefficient equation that w must satisfy. (b) Obtain a the general solution to the ODE that w satisfies and then stop.
(c) Explain very briefly why v can be obtained from w without actually integrating. ***Do not attempt to actually find v.***

Silly 10 Point Bonus: Let $f(x) = x$ and $g(x) = \sin(x)$. (a) It is trivial to obtain a 4th order homogeneous linear constant coefficient ordinary differential equation with f and g as solutions. Do so.
(b) It's only slightly messier to obtain a 2nd order homogeneous linear ordinary differential equation with $\{ f , g \}$ as a fundamental set of solutions. Do so. [Say where your work is, for it won't fit here.]