Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me the all magic on the page.

Test #:

1. (10 pts.) Locate and classify the singular points of the following second order homogeneous O.D.E. Use complete sentences to describe the type of points and where they occur.

$$x^2(x-2)^2y'' + 2(x-2)y' + (x+1)y = 0$$

2. (15 pts.) Suppose

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

is a solution of the homogeneous second order linear equation

$$y'' - y' + 2xy = 0.$$

Very neatly obtain the recurrence formula(s) needed to determine the coefficients of $y(x)$. DO NOT SPEND TIME ATTEMPTING TO GET ACTUAL COEFFICIENTS.

3. (15 pts.) (a) If $f(t)$ and $g(t)$ are piecewise continuous functions defined for $t \geq 0$, what is the definition of the convolution of $f$ with $g$, $(f*g)(t)$??

$$(f*g)(t) =$$

(b) Using only the definition of the convolution as a definite integral, not some fancy transform shenanigans, compute $(f*g)(t)$ when $f(t) = 3t^2$ and $g(t) = 4t$.

$$(f*g)(t) =$$

(c) Using the Laplace transform table, compute the Laplace transform of $f*g$ when $f(t) = t\cdot\cos(t)$ and $g(t) = \exp(2t)$. [Do not attempt to simplify the algebra after computing the transform.]

$$\mathfrak{L}\{(f*g)(t)\}(s) =$$

4. (10 pts.) (a) Suppose that $f(t)$ is defined for $t > 0$. What is the definition of the Laplace transform of $f$, $\mathfrak{L}\{f(t)\}$, in terms of a definite integral??

$$\mathfrak{L}\{f(t)\}(s) =$$

(b) Using only the definition, not the table, compute the Laplace transform of

$$f(t) = \begin{cases} 0 \ , \ \mathit{if} \ \ 0 < t < 4 \\ 3 \ , \ \mathit{if} \ \ 4 < t. \end{cases}$$

$$\mathfrak{L}\{f(t)\}(s) =$$

5. (10 pts.) The equation below has a regular singular point at $x_0 = 0$.

$$x^2y'' - xy' + (x^2+1)y = 0$$

(a) Obtain the indicial equation for the ODE at $x_0 = 0$ and its two roots. (b) Then use all the information available and Theorem 6.3 to say what the two nontrivial linearly independent solutions given by theorem look like without attempting to obtain the coefficients of the power series involved.

6. (10 pts.) Compute $f(t) = \mathcal{L}^{-1}\{F(s)\}(t)$ when

(a) $F(s) = \dfrac{7}{(2s+1)^3}$

(b) $F(s) = \dfrac{2s+12}{s^2+6s+13}$

7. (5 pts.) Circle the letter corresponding to the correct response: If $F(s) = \mathcal{L}\{t^2\sin(bt)\}(s)$ ,then $F(s) =$

(a) $\left[\dfrac{2}{s^3}\right]\cdot\left[\dfrac{b}{(s^2+b^2)}\right]$ (b) $-\dfrac{d}{ds}\left[\dfrac{2bs}{(s^2+b^2)^2}\right]$

(c) $-\dfrac{d^2}{ds^2}\left[\dfrac{b}{s^2+b^2}\right]$ (d) $\dfrac{2bs}{s^2(s^2+b^2)^2}$

(e) None of the above.

8. (15 pts.) Transform the given initial value problem into an algebraic equation in $\mathfrak{L}\{y\}$ and solve for $\mathfrak{L}\{y\}$. Do not take inverse transforms and do not attempt to combine terms over a common denominator. Be very careful.

$$3y''(t) - 5y'(t) + 7y(t) = \sin(2t) \ ; \ y(0) = 4 \ , \ y'(0) = 6 \ .$$

9. (10 pts.) The solution to a certain linear ordinary differential equation with coefficient functions that are analytic at $x_0 = 0$ is of the form

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

where the coefficients satisfy the following equations:

$$-c_2 = 0, \quad c_0 + 3c_1 - 6c_3 = 0, \text{ and}$$

$$c_{n+2} = \frac{n(n+2)c_n + c_{n-1}}{(n+2)(n+1)} \quad \text{for all } n \geq 2.$$

Determine the exact numerical value of the coefficients $c_0$, $c_1$, $c_2$, $c_3$, and $c_4$ for the particular solution that satisfies the initial conditions $y(0) = 0$ and $y'(0) = 1$.

$c_0$ =

$c_1$ =

$c_2$ =

$c_3$ =

$c_4$ =

**Silly 10 Point Bonus:** Write the function

$$f(t) = \cos^5(bt)$$

as a linear combination of sine and/or cosines actually appearing in the Laplace transform table provided. Say where your work is, for it won't fit here.