
General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Write complete sentences, and use notation correctly. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate. Eschew obfuscation.*

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1. (80 pts.) Solve each of the following differential equations or initial value problems. If there is no initial condition, obtain the general solution. [20 points/part]

(a) $\frac{dy}{dx} + 4xy = 8xy^{-3}$; $y(0) = 2$ This is obviously a Bernoulli equation as written. A little bit of algebra, though, converts the ODE into one that may be seen as separable

$$\frac{dy}{dx} = 8xy^{-3} - 4xy = 4x\left(\frac{2 - y^4}{y^3}\right)$$

with a real constant solution consisting of

$$y(x) = 2^{\frac{1}{4}} .$$

Obviously this constant solution does not satisfy the initial condition.

Separating variables algebraically leads to

$$4x\,dx + \left(\frac{y^3}{y^4 - 2}\right)dy = 0 .$$

Integrating this yields

$$\int 4x\,dx + \int\left(\frac{y^3}{y^4 - 2}\right)dy = C ,$$

or

$$2x^2 + \frac{1}{4}\ln|y^4 - 2| = C$$

as a one-parameter family of implicit solutions to the ODE.

Applying the initial condition allows us to determine the constant *C*. In fact,

$$C = \frac{1}{4}\ln(14) = \ln(14^{\frac{1}{4}}) .$$

Thus, an implicit solution to the IVP is given by

$$2x^2 + \frac{1}{4}\ln|y^4 - 2| = \ln(14^{\frac{1}{4}}) .$$

You may do a little additional algebra to obtain the implicit solution

$$y^4 = 2 + 14e^{-8x^2} .$$

An explicit solution: $y(x) = (2 + 14e^{-8x^2})^{\frac{1}{4}}$