Student Number: **Exam Number:**

General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Make your arguments and proofs as complete as possible. Be very very careful. Read each question twice and provide an answer to exactly what is requested. Remember that what is illegible or incomprehensible is worthless. Good Luck!*

1. (5 pts.) What value(s) of the Boolean variables x and y satisfy $x + xy = xy$? [Hint: Build a table.]

2. (5 pts.) Find the sum-of-products expansion for the function Boolean function F defined by $F(x,y) = x + y$.

3. (5 pts.) Write the contrapositive and converse of the statement, "If dogs have wings, then fur will fly," and label unambiguously. Which is equivalent to the original statement?

4. (5 pts.) Give a recursive definition of the sequence $\{a_n\}$, where n = 1,2,3,.. if $a_n = 7n + 1$. [Be very careful of the quantification used in the induction step.]

Basis Step:

Induction Step:

5. (5 pts.) Let $f:\mathbb{Z} \to \mathbb{Z}$ be the function defined by the formula $f(n) = 2n - 1$. Prove f is one-to-one.

6. (5 pts.) Give a recursive definition of the set S of positive integers that are multiples of 4. [Be very careful of the quantification used in the induction step.]

Basis Step:

Induction Step:

7. (10 pts.) A club has 50 members with 20 males and 30 females. Show how to do the computation that answers each of the following questions.

(a) How many ways are there to choose committees consisting of four members of the club?

(b) How many ways are there to choose a president, vice president, secretary, and treasurer of the club?

(c) How many ways are there to choose committees consisting of four people if exactly three of the four must be female?

(d) How many ways are there to choose committees consisting of four people if none of them can be female?

(e) How many ways are there to choose committees consisting of four people if at least one must be female.

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8. (5 pts.) (a) The following proposition represents an invalid argument form: $[\neg p \wedge (p \rightarrow q)] \rightarrow \neg q$.

What is the name of the popular fallacy given by the proposition above?

(b) The following proposition represents an invalid argument form: $[q \wedge (p \rightarrow q)] \rightarrow p$.

What is the name of the popular fallacy given by the proposition above?

(c) Why is reasoning using either of the argument forms found in (a) or (b) deemed invalid?

9. (5 pts.) Label each of the following assertions with "true" or "false". **Be sure to write out the entire word.**

(a) The set of all subsets of the natural numbers is countable.

(b) The number of functions from a set with 8 elements to a set with 15 elements is P(15,8).

(c) The coeffient of x^4y^5 in the expansion of $(x + y)^9$ is the number of 5 element subsets of an 9 element set.

(d) There is a one-to-one correspondence between the natural numbers and the rational numbers.

(e) The total number of ways to assign truth values to five true-false problems by using the letters *T* and *F* is given by the sum below.

$$\sum_{k=1}^{5} C(5,k)$$

10. (5 pts.) Suppose that A and B are sets. Prove the following using the definition of the terms *subset* and *intersection*. If $A = A \cap B$, then $A \subseteq B$.

11. (5 pts.) What is the minimum number of students required in your Discrete Mathematics class to be sure that at least 6 have birthdays occurring in the same month this year? Explain.

12. (10 pts.) Suppose $A = \{\varnothing, 3, 4\}$ and $B = \{\varnothing, 3, \{\varnothing\}\}$. Then

$A \cap B =$

$A \times B =$

$|P(A)| =$

$A - B =$

$A \cup B =$

13. (5 pts.) Suppose that R is an equivalence relation on a nonempty set A. Recall that for each a ε A, the equivalence class of a is the set $[a] = \{s \mid (a,s) \in R\}$. Prove the following proposition: If $[a] \cap [b] \neq \emptyset$, then $[a] = [b]$.

Hint: The issue is the set equality, $[a] = [b]$, under the hypothesis that $[a] \cap [b] \neq \emptyset$. So pretend $[a] \cap [b] \neq \emptyset$ and use this to show $s \in [a] \rightarrow s \in [b]$, and $s \in [b] \rightarrow s \in [a]$. Be explicit regarding your use of the relational properties of R.

14. (5 pts.) Let $\{a_n\}$ be defined by the formula $a_n = 4n + 2$ for $n = 1,2,3,\ldots.$ The sequence $\{b_n\}$ is defined recursively by $b_1 = 6$ and $b_{n+1} = b_n + 4$ for $n = 1,2,3,\ldots.$ Give a proof by induction that $a_n = b_n$ for $n = 1,2,3,\ldots.$

15. (5 pts.) Construct the ordered rooted binary tree representing the following expression:

$$(C - (A \cap B)) = ((C - A) \cup (C - B))$$

16. (5 pts.) Suppose $G_1 = (V_1, E_1)$ is an undirected graph with adjacency matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix},$$

and $G_2 = (V_2, E_2)$ is an undirected graph with adjacency matrix

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

Are G_1 and G_2 isomorphic?? Either display an explicit graph isomorphism $g: V_1 \to V_2$ or give an invariant that one graph has but the other doesn't have. [Warning: The graphs are obviously *not* simple. Drawing them might help.]

17. (10 pts.) The following is a valid argument:

"Each of five friends, Larry, Moe, Curly, Samantha, and Anitra, has taken a course in discrete mathematics and passed with a grade of at least a C. Every student who has taken and passed a discrete math class with a grade of at least a C can take a course in algorithms. Therefore, all five friends can take a course in algorithms."

The validity of the argument can be seen easily by symbolizing the argument using propositional functions and quantifiers as follows:

Define propositional functions as follows:

f(x) : "x is one of the friends listed."
d(x) : "x has taken and passed discrete math with at least a C."
a(x) : "x can take a course in algorithms."

Then the argument translates into this:

$$\begin{array}{l}(\forall x)(f(x) \rightarrow d(x))\\ (\forall x)(d(x) \rightarrow a(x))\\ \hline \therefore (\forall x)(f(x) \rightarrow a(x))\end{array}$$

In the proof of validity which follows provide the justification(s) for each step. In doing this, you should explicitly cite rules of inference and quantification, hypotheses, and preceding steps by number.

1. $(\forall x)(f(x) \rightarrow d(x))$ Justification:

2. $f(y) \rightarrow d(y)$ Justification:

3. $(\forall x)(d(x) \rightarrow a(x))$ Justification:

4. $d(y) \rightarrow a(y)$ Justification:

5. $f(y) \rightarrow a(y)$ Justification:

6. $(\forall x)(f(x) \rightarrow a(x))$ Justification: