

Rules of Inference and Quantification

Rule of Inference	Tautology	Name
$\frac{p}{\therefore p \vee q}$	$p \rightarrow (p \vee q)$	Addition
$\frac{p \wedge q}{\therefore p}$	$(p \wedge q) \rightarrow p$	Simplification
$\frac{p}{q} \quad \frac{q}{\therefore p \wedge q}$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$\frac{p}{p \rightarrow q} \quad \frac{p \rightarrow q}{\therefore q}$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus Ponens
$\frac{\neg q}{p \rightarrow q} \quad \frac{p \rightarrow q}{\therefore \neg p}$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus Tollens
$\frac{p \rightarrow q}{q \rightarrow r} \quad \frac{q \rightarrow r}{\therefore p \rightarrow r}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical Syllogism
$\frac{p \vee q}{\neg p} \quad \frac{\neg p}{\therefore q}$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive Syllogism

Rule of Inference	Name
$\frac{\forall x P(x)}{\therefore P(c) \text{ if } c \in U}$	Universal Instantiation
$\frac{P(c) \text{ for an arbitrary } c \in U}{\therefore \forall x P(x)}$	Universal Generalization
$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c \in U}$	Existential Instantiation
$\frac{P(c) \text{ for some element } c \in U}{\therefore \exists x P(x)}$	Existential Generalization