1. (15 pts.) Construct the truth tables for each of the following compound propositions in the space provided:

(a) $p \wedge q$ (b) $p \rightarrow q$ (c) $\neg p$: See pages 3 through 7.

2. (10 pts.) Write the contrapositive and converse of the statement, "If pigs have wings, then the bacon will fly," and label unambiguously. Which is equivalent to the original statement?

Contrapositive: If the bacon won't fly, then pigs don't have wings.

Converse: If the bacon will fly, then pigs have wings.

Only the contrapositive is equivalent to the original statement.

3. (15 pts.) Let F(x,y) be the statement "x can fool y". The universe of discourse is all people. Use quantifiers to express each of the following statements:

(a) Everyone can fool Frodo. $(\forall x)F(x,\text{Frodo})$

(b) No one can fool Gandalf. $\neg(\exists x)F(x,\text{Gandalf})$ or equivalently $(\forall x)\neg F(x,\text{Gandalf})$

(c) Everyone can fool someone. $(\forall x)(\exists y)F(x,y)$

4. (10 pts.) Determine the truth value of each of the following statements if the universe of discourse of each variable is the set of real numbers, $\mathbb{R}$.

(a) $(\exists x)(\forall y)(x + y = 0)$ False (b) $(\forall x)(\exists y)(x + y = 0)$ True

5. (15 pts.) Suppose $A = \{\varnothing,\{\varnothing\}\}$ and $B = \{\varnothing,3,4\}$. Then

$A \cup B = \{\varnothing, \{\varnothing\}, 3, 4\}$

$A \times B = \{ (\varnothing,\varnothing),(\varnothing,3),(\varnothing,4),(\{\varnothing\},\varnothing),(\{\varnothing\},3),(\{\varnothing\},4)\}$

$|P(A)| = 2^{|A|} = 2^2 = 4$

6. (10 pts.) What can you say about sets A and B if $B = A \cup B$? Prove your assertion. If $B = A \cup B$, then $A \subseteq B$. Proof: $x \varepsilon A$ implies $x \varepsilon A$ or $x \varepsilon B$ which implies $x \varepsilon A \cup B = B$. // It turns out that the converse is also true.

7. (15 pts.) Suppose that $f:\mathbb{R} \rightarrow \mathbb{Z}$ is the function defined by the formula $f(x)=\lceil x \rceil$, and suppose that $A = \{x \varepsilon \mathbb{R} \mid -2 < x \leq 3\}$ and $B = \{x \varepsilon \mathbb{R} \mid -1 < x \leq \pi\}$. Using appropriate notation, give each of the following. $A - B = (-2,-1]$ $f(A) = \{-1,0,1,2,3\}$

$f^{-1}(\{2,3\}) = f^{-1}(\{2\}) \cup f^{-1}(\{3\}) = (1,2] \cup (2,3] = (1,3]$

8. (10 pts.) Suppose $g:A \rightarrow B$ and $f:B \rightarrow C$ are functions. Prove that if both f and g are injective, then $f \circ g:A \rightarrow C$ is injective.
Proof: [1.6,19a]