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1. (15 pts.) Suppose matrices A, B, and C are as given below.

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 & 8 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 4 & 1 \\ 3 & -2 \\ 2 & 1 \\ 1 & -2 \end{bmatrix}$$

Then

$$\mathbf{B} + \mathbf{C} = \begin{bmatrix} 5 & -1 \\ 4 & -1 \\ 3 & -1 \\ 2 & -1 \end{bmatrix} \qquad \mathbf{AB} = \begin{bmatrix} 7 & 4 \\ -6 & -9 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{C}^t = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & -2 & 1 & -2 \end{bmatrix}$$

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2. (10 pts.) A club has 50 members.
(a) There are C(50,9) ways to choose nine members of the club to serve on an executive committee.
(b) There are P(50,4) ways to choose a president, vice president, secretary, and treasurer of the club.

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3. (15 pts.) (a) Suppose f is a function defined recursively by f(0) = 1 and f(n+1) = f(n) + 5 for n = 0,1,2,3,.... Then f(1) = 6, f(2) = 11, f(3) = 16, and f(4) = 21.

(b) Give a recursive definition of the sequence $\{a_n\}$, where n = 1,2,3,.. if $a_n = 2n + 1$. [Be very careful of the quantification used in the induction step.] Basis Step: $a_1 = 3$
Induction Step: $(\forall n)(n \varepsilon \mathbb{N}^+ \rightarrow a_{n+1} = a_n + 2)$ or $(\forall n \varepsilon \mathbb{N}^+)(a_{n+1} = a_n + 2)$.

(c) Give a recursive definition of the set S of positive integers that are multiples of 3. Basis Step: $3 \ \varepsilon \ S$
Induction Step: $(\forall x)(x \ \varepsilon \ S \rightarrow x+3 \ \varepsilon \ S)$ or alternatively
$(\forall x)(\forall y)((x \ \varepsilon \ S \wedge y \ \varepsilon \ S) \rightarrow x+y \ \varepsilon \ S)$

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4. (10 pts.) Let $\{a_n\}$ be defined by the formula $a_n = 5n + 2$ for n = 1,2,3,.... Define the sequence $\{b_n\}$ recursively by $b_1 = 7$ and $b_{n+1} = b_n + 5$ for n = 1,2,3,.... Give a proof by induction that $a_n = b_n$ for n = 1,2,3,....
Proof:
Basis Step: Evidently, $a_1 = 5(1) + 2 = 7 = b_1$.

Induction Step: To show $(\forall n \varepsilon \mathbb{N}^+)(a_n = b_n \rightarrow a_{n+1} = b_{n+1})$, assume $a_n = b_n$ for an arbitrary $n \varepsilon \mathbb{N}^+$. Then using the induction hypothesis,

$a_n = b_n \rightarrow a_n + 5 = b_n + 5$	[Algebra; Motivation: Use the recursive definition of $\{b_n\}$.]
$\rightarrow (5n + 2) + 5 = b_{n+1}$	[Recursive definition of $\{b_n\}$, definition of $\{a_n\}$]
$\rightarrow 5(n+1) + 2 = b_{n+1}$	[Algebra; Motivation: Use the definition of $\{a_n\}$]
$\rightarrow a_{n+1} = b_{n+1}$	[The definition of $\{a_n\}$]

Since we have shown that $a_n = b_n$ implies $a_{n+1} = b_{n+1}$ for an arbitrary $n \varepsilon \mathbb{N}^+$, universal generalization implies the truth of $(\forall n \varepsilon \mathbb{N}^+)(a_n = b_n \rightarrow a_{n+1} = b_{n+1})$.

Finally, since we have verified the hypotheses of the Principle of Mathematical induction, using modus ponens, we may conclude that the proposition $(\forall n \varepsilon \mathbb{N}^+)(a_n = b_n)$ is true.

5. (15 pts.) Label each of the following assertions with "true" or "false". **Be sure to write out the entire word.**
(a) The set of all subsets of the natural numbers is countable. False.
(b) The number of one-to-one functions from a set with 8 elements to a set with 15 elements is P(15,8). True.
(c) The coeffient of x^3y^5 in the expansion of $(x + y)^8$ is the number of 3 element subsets of an 8 element set. True.
(d) There is a one-to-one correspondence between the natural numbers and the real numbers in the interval (0,1). False.
(e) The total number of ways to assign truth values to five true-false problems by using the letters *T* and *F* is given by the sum below. True.

$$\sum_{k=0}^{5} C(5,k)$$

6. (10 pts.) What is the minimum number of students required in your Discrete Mathematics class to be sure that at least 4 have birthdays occurring on the same day of the week this year? Explain. From the generalized pigeonhole principle, we need to find the smallest positive integer N so that $\lceil N/7 \rceil = 4$. Thus, N = (3*7) + 1 = 22 will do.

7. (10 pts.) (10 pts.) The following proposition represents an invalid argument form: $[\neg p \wedge (p \to q)] \to \neg q$.
(a) What is the name of the popular fallacy given by the proposition above? This is the infamous fallacy of denying the hypothesis. [Some logic texts call this the fallacy of denying the antecedent.]
(b) Why is reasoning using this argument form deemed invalid?
Since the proposition above is a contingency, one cannot be one hundred per cent certain that the truth of the conclusion follows from the truth of the hypothesis. That, of course, is unacceptable behavior for a rule of inference, and is why we require all the implications used as rules of inference to be tautologies. [Think about betting your life on the rules of inference!!]

8. (15 pts.)
c(Frodo)
$\neg$h(Frodo)
$(\forall x)(j(x) \to h(x))$

$\therefore (\exists x)(c(x) \wedge \neg j(x))$

Proof of validity:

1. $\neg$h(Frodo) : Hypothesis
2. $(\forall x)(j(x) \to h(x))$: Hypothesis
3. $j(Frodo) \to h(Frodo)$: 2,Universal Instantiation
4. $\neg$j(Frodo) : 1,3,Modus Tollens
5. c(Frodo) : Hypothesis
6. $c(Frodo) \wedge \neg j(Frodo)$: 5,4,Conjunction
7. $(\exists x)(c(x) \wedge \neg j(x))$: 6,Existential Generalization