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**General directions:** Read each problem carefully and do exactly what is requested. Show all your work neatly. Use complete sentences and use notation correctly. Make your arguments and proofs as complete as possible. Remember that what is illegible or incomprehensible is worthless.

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1. (15 pts.) Suppose matrices A, B, and C are as given below.

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 & 8 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 4 & 1 \\ 3 & -2 \\ 2 & 1 \\ 1 & -2 \end{bmatrix}$$

Then

$$\mathbf{B} + \mathbf{C} =$$

$$\mathbf{AB} =$$

$$\mathbf{C}^t =$$

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2. (10 pts.) A club has 50 members. Show how to do the computation that answers each of the following questions.

(a) How many ways are there to choose nine members of the club to serve on an executive committee?

(b) How many ways are there to choose a president, vice president, secretary, and treasurer of the club?

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3. (15 pts.) (a) Suppose  $f$  is a function defined recursively by  $f(0) = 1$  and  $f(n+1) = f(n) + 5$  for  $n = 0, 1, 2, 3, \dots$ . Then

$$f(1) =$$

$$f(2) =$$

$$f(3) =$$

$$f(4) =$$

(b) Give a recursive definition of the sequence  $\{a_n\}$ , where  $n = 1, 2, 3, \dots$  if  $a_n = 2n + 1$ . [Be very careful of the quantification used in the induction step.]

Basis Step:

Induction Step:

(c) Give a recursive definition of the set  $S$  of positive integers that are multiples of 3.

Basis Step:

Induction Step:

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4. (10 pts.) Let  $\{a_n\}$  be defined by the formula  $a_n = 5n + 2$  for  $n = 1, 2, 3, \dots$ . Define the sequence  $\{b_n\}$  recursively by  $b_1 = 7$  and  $b_{n+1} = b_n + 5$  for  $n = 1, 2, 3, \dots$ . Give a proof by induction that  $a_n = b_n$  for  $n = 1, 2, 3, \dots$ .

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5. (15 pts.) Label each of the following assertions with "true" or "false". **Be sure to write out the entire word.**

(a) The set of all subsets of the natural numbers is countable.

(b) The number of one-to-one functions from a set with 8 elements to a set with 15 elements is  $P(15,8)$ .

(c) The coefficient of  $x^3y^5$  in the expansion of  $(x + y)^8$  is the number of 3 element subsets of an 8 element set.

(d) There is a one-to-one correspondence between the natural numbers and the real numbers in the interval  $(0,1)$ .

(e) The total number of ways to assign truth values to five true-false problems by using the letters  $T$  and  $F$  is given by the sum below.

$$\sum_{k=0}^5 C(5,k)$$

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6. (10 pts.) What is the minimum number of students required in your Discrete Mathematics class to be sure that at least 4 have birthdays occurring on the same day of the week this year? Explain.

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7. (10 pts.) (10 pts.) The following proposition represents an invalid argument form:  $[\neg p \wedge (p \rightarrow q)] \rightarrow \neg q$ .

(a) What is the name of the popular fallacy given by the proposition above?

(b) Why is reasoning using this argument form deemed invalid?

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8. (15 pts.) The following is a valid argument:

"Frodo, a student in this class, cannot get a high-paying job. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class does not know how to write programs in JAVA."

The validity of the argument can be seen easily by symbolizing the argument using propositional functions and quantifiers as follows:

Define propositional functions as follows:

$c(x)$  : "x is in this class"  
 $j(x)$  : "x knows how to write programs in JAVA"  
 $h(x)$  : "x can get a high-paying job"

Then the argument translates into this:

$c(\text{Frodo})$   
 $\neg h(\text{Frodo})$   
 $(\forall x)(j(x) \rightarrow h(x))$   


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 $\therefore (\exists x)(c(x) \wedge \neg j(x))$

In the proof of validity which follows provide the justification(s) for each step. In doing this you should explicitly cite rules of inference and quantification, hypotheses, and preceding steps by number.

1.  $\neg h(\text{Frodo})$  Justification:
2.  $(\forall x)(j(x) \rightarrow h(x))$  Justification:
3.  $j(\text{Frodo}) \rightarrow h(\text{Frodo})$  Justification:
4.  $\neg j(\text{Frodo})$  Justification:
5.  $c(\text{Frodo})$  Justification:
6.  $c(\text{Frodo}) \wedge \neg j(\text{Frodo})$  Justification:
7.  $(\exists x)(c(x) \wedge \neg j(x))$  Justification: