1. (15 pts.) Suppose matrices A, B, and C are as given below.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & 8 \\ 0 & 0 & 1 & -7 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ 1 & 1 \\ -2 & 1 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 4 \\ -2 & 3 \\ 1 & 2 \\ -2 & 1 \end{bmatrix}$$

Then

$$\mathbf{B} + \mathbf{C} = \begin{bmatrix} -1 & 5 \\ -1 & 4 \\ -1 & 3 \\ -1 & 2 \end{bmatrix} \qquad \mathbf{AB} = \begin{bmatrix} 0 & 0 \\ 4 & 7 \\ -9 & -6 \end{bmatrix} \qquad \mathbf{C}^t = \begin{bmatrix} 1 & -2 & 1 & -2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

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2. (10 pts.) A club has 75 members.
(a) There are C(75,10) ways to choose ten members of the club to serve on an executive committee.
(b) There are P(75,4) ways to choose a president, vice president, secretary, and treasurer of the club.

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3. (15 pts.) (a) Suppose f is a function defined recursively by $f(0) = 1$ and $f(n+1) = f(n) + 3$ for $n = 0,1,2,3,\ldots$. Then $f(1) = 4$, $f(2) = 7$, $f(3) = 10$, and $f(4) = 13$.

(b) Give a recursive definition of the sequence $\{a_n\}$, where $n = 1,2,3,..$ if $a_n = 5n + 1$. [Be very careful of the quantification used in the induction step.] Basis Step: $a_1 = 6$
Induction Step: $(\forall n)(n \varepsilon \mathbb{N}^+ \rightarrow a_{n+1} = a_n + 5)$ or $(\forall n \varepsilon \mathbb{N}^+)(a_{n+1} = a_n + 5)$.

(c) Give a recursive definition of the set S of positive integers that are multiples of 4. Basis Step: $4 \varepsilon S$
Induction Step: $(\forall x)(x \varepsilon S \rightarrow x+4 \varepsilon S)$ or alternatively $(\forall x)(\forall y)((x \varepsilon S \wedge y \varepsilon S) \rightarrow x+y \varepsilon S)$

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4. (10 pts.) Let $\{a_n\}$ be defined by the formula $a_n = 3n + 2$ for $n = 1,2,3,\ldots$. Define the sequence $\{b_n\}$ recursively by $b_1 = 5$ and $b_{n+1} = b_n + 3$ for $n = 1,2,3,\ldots$. Give a proof by induction that $a_n = b_n$ for $n = 1,2,3,\ldots$.
Proof:
Basis Step: Evidently, $a_1 = 3(1) + 2 = 5 = b_1$.

Induction Step: To show $(\forall n \varepsilon \mathbb{N}^+)(a_n = b_n \rightarrow a_{n+1} = b_{n+1})$, assume $a_n = b_n$ for an arbitrary $n \varepsilon \mathbb{N}^+$. Then using the induction hypothesis,

$a_n = b_n \rightarrow a_n + 3 = b_n + 3$	[Algebra; Motivation: Use the recursive definition of $\{b_n\}$.]
$\rightarrow (3n + 2) + 3 = b_{n+1}$	[Recursive definition of $\{b_n\}$, definition of $\{a_n\}$]
$\rightarrow 3(n+1) + 2 = b_{n+1}$	[Algebra; Motivation: Use the definition of $\{a_n\}$]
$\rightarrow a_{n+1} = b_{n+1}$	[The definition of $\{a_n\}$]

Since we have shown that $a_n = b_n$ implies $a_{n+1} = b_{n+1}$ for an arbitrary $n \varepsilon \mathbb{N}^+$, universal generalization implies the truth of $(\forall n \varepsilon \mathbb{N}^+)(a_n = b_n \rightarrow a_{n+1} = b_{n+1})$.

Finally, since we have verified the hypotheses of the Principle of Mathematical induction, using modus ponens, we may conclude that the proposition $(\forall n \varepsilon \mathbb{N}^+)(a_n = b_n)$ is true.

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5. (15 pts.) Label each of the following assertions with "true" or "false". **Be sure to write out the entire word.**

(a) The set of all subsets of the natural numbers is not countable. True.

(b) The number of functions from a set with 8 elements to a set with 15 elements is C(15,8). False.

(c) The coeffient of x^3y^5 in the expansion of $(x + y)^8$ is the number of 3-permutations of an 8 element set. False.

(d) There is a one-to-one correspondence between the real numbers in the interval (-200,200) and the real numbers in the interval (0,1). True.

(e) The total number of ways to assign truth values to five true-false problems by using the letters *T* and *F* is given by the sum below. False.

$$\sum_{k=1}^{5} C(5,k)$$

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6. (10 pts.) What is the minimum number of students required in your Discrete Mathematics class to be sure that at least 4 have birthdays occurring in the same month this year? Explain. From the generalized pigeonhole principle, we need to find the smallest positive integer N so that $\lceil N/12 \rceil = 4$. Thus, N = (3*12) + 1 = 37 will do.

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7. (10 pts.) The following proposition represents an invalid argument form: $[q \wedge (p \rightarrow q)] \rightarrow p$.

(a) What is the name of the popular fallacy given by the proposition above? This is the infamous fallacy of affirming the conclusion. [Some logic texts call this the fallacy of affirming the consequent.]

(b) Why is reasoning using this argument form deemed invalid?

Since the proposition above is a contingency, one cannot be one hundred per cent certain that the truth of the conclusion follows from the truth of the hypothesis. That, of course, is unacceptable behavior for a rule of inference, and is why we require all the implications used as rules of inference to be tautologies. [Think about betting your life on the rules of inference!!]

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8. (15 pts.)

$$\begin{array}{l} c(\text{Frodo}) \\ j(\text{Frodo}) \\ (\forall x)(j(x) \rightarrow h(x)) \\ \hline \therefore (\exists x)(c(x) \wedge h(x)) \end{array}$$

Proof of validity:

1.	$(\forall x)(j(x) \rightarrow h(x))$	: Hypothesis
2.	$j(\text{Frodo}) \rightarrow h(\text{Frodo})$	: 1,Universal Instantiation
3.	$j(\text{Frodo})$	: Hypothesis
4.	$h(\text{Frodo})$	: 2,3,Modus Ponens
5.	$c(\text{Frodo})$	: Hypothesis
6.	$c(\text{Frodo}) \wedge h(\text{Frodo})$	: 5,4,Conjunction
7.	$(\exists x)(c(x) \wedge h(x))$	: 6,Existential Generalization