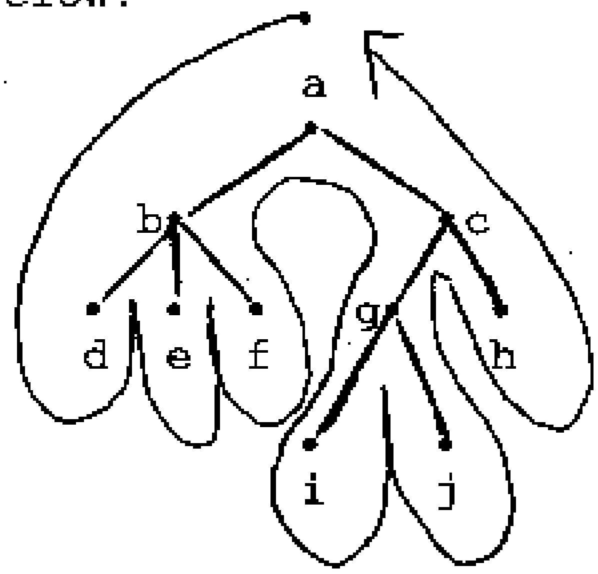
General directions: Read each problem carefully and do exactly what is requested. Show all your work neatly. Use complete sentences and use notation correctly. Make your arguments and proofs as complete as possible. Remember that what is illegible or incomprehensible is worthless.

1. (15 pts.) Given the ordered rooted tree below, list the vertices in the order in which they are visited in (a) a preorder traversal of the tree, (b) an inorder traversal of the tree, and (c) a postorder traversal of the tree. Place your lists in the appropriate place below.

17, 18 April.



(a) preorder traversal : a b d e f c g i j h

(b) inorder traversal : d b e f a i g j c h

(c) postorder traversal : d e f b i j g h c a

2. (10 pts.) (a) What is the numerical value of the prefix expression below?

$\uparrow - * 3\ 3 (* 4\ 2) 5 = \uparrow - (* 3\ 3) 8\ 5 = \uparrow (- 9\ 8) 5$
$= \uparrow 1\ 5 = 1$

(b) What is the numerical value of the postfix expression below?

$(9\ 3\ /) 5 + 7\ 2 - * = (3\ 5 +) 7\ 2 - *$
$= 8 (7\ 2 -) * = 8\ 5 *$
$= 40$

3. (15 pts.) Suppose R and S are relations on the set A = {a,b,c} represented by the matrices given below.

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

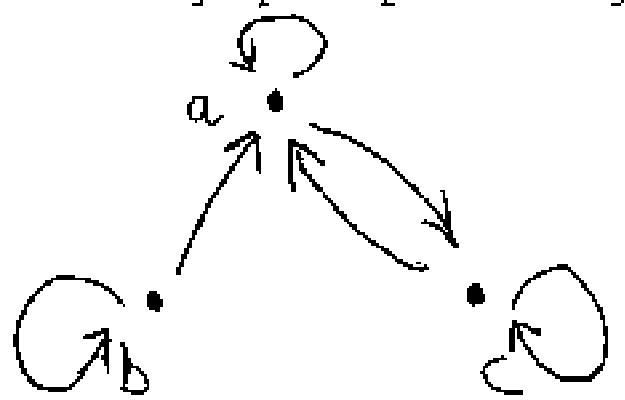
(a) If we assume that the matrices were constructed using the order listed above for the set A, then

R = $\{(a,a), (a,b), (b,a), (b,c), (c,b)\}$

(b) What is the matrix representing $R \cap S$?

$$M_{R \cap S} = M_R \wedge M_S = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(c) Construct the digraph representing S in the space below.

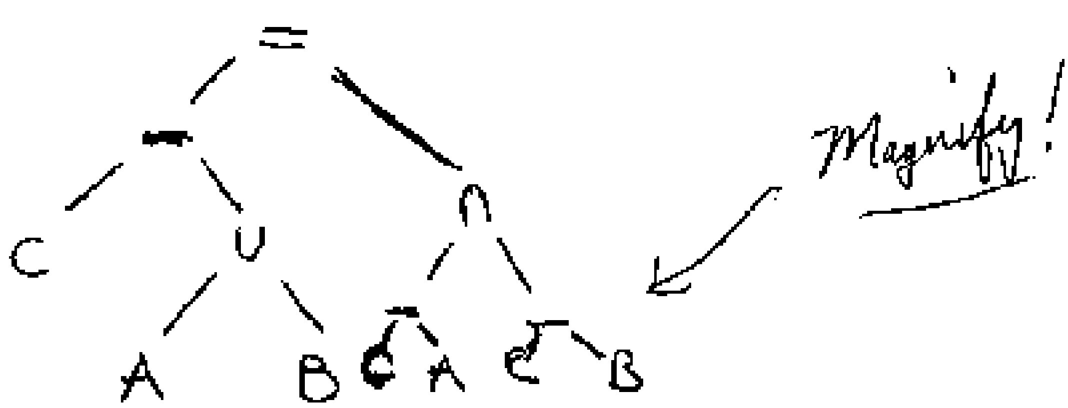


4. (10 pts.) (a) Using the prefix code a:001, b:0001, e:1, r:0000, s:0100, t:011, and x:01010, decode the following bit string:

0111001000100100000100

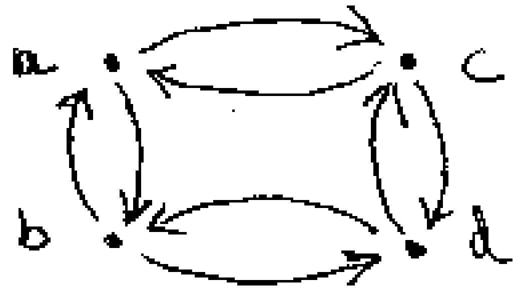
t e a b a r s

(b) Construct the ordered rooted binary tree representing the following expression: $[C - (A \cup B)] = [(C - A) \cap (C - B)]$ ← Give in Class

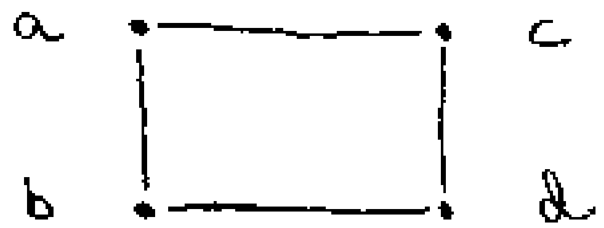


5. (15 pts.) (a) Draw a directed graph G_1 whose adjacency matrix is

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

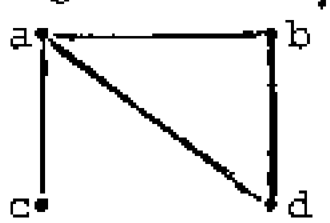


(b) Now draw an undirected graph G_2 which is represented by the adjacency matrix in part (a) of this problem.



(c) Is $G_2 = (V_2, E_2)$, above, isomorphic to the simple graph $G_3 = (V_3, E_3)$ given below? Either display an isomorphism $f: V_2 \to V_3$ or very briefly explain why there is no such function by revealing an invariant that one graph has that the other doesn't.

No, G_2 is not isomorphic to G_3. There is no such function since G_3 has a vertex of degree 3 and all of G_2's vertices are of degree 2.



[There are other invariants that can be used here!]

6. (10 pts.) Provide a proof by induction that $2^n \ge 2n$ for every positive integer n. Be explicit concerning your use of the induction hypothesis in the induction step.

Basis: $2^1 = 2 = 2\cdot 1$. Thus $2^1 \ge 2\cdot 1$.

Induction Step: Will show $(\forall n)(n \in N^+ \text{ and } 2^n \ge 2n \rightarrow 2^{n+1} \ge 2(n+1))$

Suppose $n \in N^+$ and $2^n \ge 2n$. Then, multiplying both sides of this inequality by 2, it follows that $2^{n+1} \ge 2(2n)$, or $2^{n+1} \ge 4n$. Now $n \ge 1 \rightarrow 2n \ge 2 \rightarrow 4n = 2n + 2n \ge 2n + 2 = 2(n+1)$. So $2^{n+1} \ge 4n$ and $4n \ge 2(n+1)$ $\rightarrow 2^{n+1} \ge 2(n+1)$. //

Since we have verified the hypotheses of the induction axiom, modus ponens allows us to conclude $(\forall n \in N^+)(2^n \ge 2n)$. //

Key

7. (15 pts.) (a) How many edges does a tree with 37 vertices have?

$$|E| = 36$$

(b) What is the maximum number of leaves that a binary tree of height 10 can have?

$$l \leq 2^{10}$$

(c) If a full 3-ary tree has 24 internal vertices, how many vertices does it have?

$$|V| = 3 \cdot 24 + 1$$
$$= 72 + 1 = 73$$

8. (10 pts.) Suppose that R is an equivalence relation on a nonempty set A. Recall that for each a ε A, the equivalence class of a is the set [a] = {s | (a,s) ε R}. Prove the following proposition: If (a,b) ε R, then [a] = [b].

Hint: The issue is the set equality, [a] = [b], under the hypothesis that (a,b) ε R. So pretend (a,b) ε R and use this to show s ε [a] → s ε [b], and s ε [b] → s ε [a]. Be explicit regarding your use of the relational properties of R.

Suppose $(a,b) \in R$. Under this hypothesis, we'll show $[a] \subseteq [b]$ and $[b] \subseteq [a]$, and thus, $[a]=[b]$.

$[a] \subseteq [b]$: $s \in [a] \rightarrow (a,s) \in R$. $(a,b) \in R$ and R symmetric $\rightarrow (b,a) \in R$. $(b,a) \in R$, $(a,s) \in R$, and R transitive $\rightarrow (b,s) \in R$. $(b,s) \in R$ and the definition of $[b] \rightarrow s \in [b]$. $\therefore s \in [a] \rightarrow s \in [b]$ for arbitrary $s \in A$. $\therefore [a] \subseteq [b]$.

$[b] \subseteq [a]$: (Easier!). $s \in [b] \rightarrow (b,s) \in R$. $(a,b) \in R$, $(b,s) \in R$ and R transitive $\rightarrow (a,s) \in R$. $(a,s) \in R$ and the definition of $[a] \rightarrow s \in [a]$. $\therefore s \in [b] \rightarrow s \in [a]$ for arbitrary $s \in A$. $\therefore [b] \subseteq [a]$. //