Student Number: **Exam Number:**

Read Me First: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Make your arguments and proofs as complete as possible. Be very careful. Read each question twice and provide an answer to exactly what is requested. Remember that what is illegible or incomprehensible is worthless. Good Luck!*

1. (10 pts.) What value(s) of the Boolean variables x and y satisfy x + xy = x + y? [Hint: Build a table.]

2. (10 pts.) Find the sum-of-products expansion for the function Boolean function F defined by F(x,y) = x + xy.

3. (10 pts.) Write the contrapositive and converse of the statement, "If dogs have wings, then fur will fly," and label unambiguously. Which is equivalent to the original statement?

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4. (10 pts.) Give a recursive definition of the sequence $\{a_n\}$, where n = 1,2,3,.. if $a_n = 4n - 2$. [Be very careful of the quantification used in the induction step.]

Basis Step:

Induction Step:

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5. (10 pts.) Let $f:\mathbb{Z} \to \mathbb{Z}$ be the function defined by the formula $f(n) = 2n - 1$.

(a) Prove f is an injection.

(b) Is f a surjection? Proof??

(c) What is the range of f?

(d) What two infinite sets are revealed to have the same cardinality?

(e) Are these sets countable?

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6. (10 pts.) Give a recursive definition of the set S of positive integers that are not divisible by 5. [Be very careful of the quantification used in the induction step.]

Basis Step:

Induction Step:

7. (20 pts.) A club has 50 members with 40 males and 10 females. Show how to do the computation that answers each of the following questions.

(a) How many ways are there to choose committees consisting of six members of the club?

(b) How many ways are there to choose a president, vice president, secretary, treasurer, and parliamentarian of the club?

(c) How many ways are there to choose committees consisting of four people if exactly three of the four must be female?

(d) How many ways are there to choose committees consisting of four people if none of them can be female?

(e) How many ways are there to choose committees consisting of four people if at least one must be female.

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8. (10 pts.) (a) The following is an invalid argument:

"If a student has taken and passed a discrete math class with a grade of at least a C, then the student may take a course in algorithms. Frodo has taken a course in algorithms. Thus Frodo has take and passed a discrete math class.'

What is the name of the popular fallacy exemplified by this argument?

(b) The following is an invalid argument:

"If $x = 5$, then $x^2 = 25$. $x \neq 5$. Thus $x^2 \neq 25$."

What is the name of the popular fallacy that this argument exemplifies?

9. (10 pts.) Label each of the following assertions with "true" or "false". **Be sure to write out the entire word.**

(a) The set of all integers that have a remainder of 2 when divided by 5 is countable.

(b) The number of one-to-one functions from a set with 15 elements to a set with 8 elements is P(15,8).

(c) The coefficient of x^5y^4 in the expansion of $(x + y)^9$ is the number of 4 element subsets of an 9 element set.

(d) There is a one-to-one correspondence between the natural numbers and the irrational numbers.

(e) The total number of ways to assign truth values to five true-false problems by using the letters *T* and *F* with at least four of the answers being true is given by the sum below.

$$\sum_{k=0}^{1} C(5,k)$$

10. (10 pts.) Suppose that A and B are sets. Give an element-wise proof of the following using the definition of the terms *subset* and *union*.

If $A \subseteq B$, then $A \cup B \subseteq B$.

11. (10 pts.) What is the minimum number of students required in your Discrete Mathematics class to be sure that at least 3 have birthdays occurring in the same month this year? Explain.

12. (20 pts.) A standard deck of playing cards consists of 52 cards divided into 4 suits of 13 cards each. There are 2 black suits, spades and clubs, and there are 2 red suits, hearts and diamonds. Each suit consists of 10 spot cards ranging from 1 or ace through 10, and 3 face cards: jack, queen, and king. A hand in bridge consists of 13 cards.

For each of the following questions, either provide an appropriate integer **or** indicate unambiguously how your answer would be computed **or** explain why the given assertion is true. [Correct notation involving combinations or permutations may be used.]

(a) How many bridge hands are there?

n =

(b) How many ways can Frodo arrange the cards in his bridge hand??

n =

(c) How many bridge hands have exactly 3 cards of one kind, e.g., 3 kings, or 3 aces, or 3 fives etc., with the remaining cards in the hand distinct and different in kind so there are no pairs??

n =

(d) How many bridge hands have exactly 6 distinct pairs?

n =

(e) Every bridge hand has at least four cards from the same suit. Why? What magic principle applies here??

13. (10 pts.) The set A = {1,2,3,4,5,6} may be partitioned into the following sets: {1}, {2,3,6}, {4}, {5}.

(a) What is the equivalence relation R defined by this partition?

(b) Draw a digraph representing this relation below.

(c) Explicitly list all the elements of the following set:

$[2]_R$ =

14. (10 pts.) Use mathematical induction to show that if n is a positive integer, then

$$\sum_{i=1}^{n} 2i = n(n+1) \ .$$

15. (10 pts.) Construct the ordered rooted binary tree representing the following identity:

$$((C - A) \cap (C - B)) = (C - (A \cup B))$$

16. (10 pts.) Suppose $G_1 = (V_1, E_1)$ is an undirected graph with adjacency matrix

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix},$$

and $G_2 = (V_2, E_2)$ is an undirected graph with adjacency matrix

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

Are G_1 and G_2 isomorphic?? Either display an explicit graph isomorphism $g: V_1 \to V_2$ or display an invariant that one graph has but the other doesn't have. [Hint: It may help to draw them.]

17. (20 pts.) The following is a valid argument:

"At least one of five friends, Larry, Moe, Curly, Samantha, and Anitra, is not permitted to take a course in algorithms. Every student who has taken and passed a discrete math class with a grade of at least a C is permitted to take a course in algorithms. Therefore, at least one of the friends has failed to take a course in discrete mathematics and pass with a grade of at least a C."

The validity of the argument can be seen easily by symbolizing the argument using propositional functions and quantifiers as follows:

Define propositional functions as follows:

f(x) : "x is one of the friends listed."
d(x) : "x has taken and passed discrete math with at least a C."
a(x) : "x is permitted to take a course in algorithms."

Then the argument translates into this:

$(\exists x)(f(x) \wedge \neg a(x))$
$(\forall x)(d(x) \rightarrow a(x))$

$\therefore (\exists x)(f(x) \wedge \neg d(x))$

In the proof of validity which follows provide the missing justification(s) for each step. You should explicitly cite rules of inference and quantification, hypotheses, and preceding steps by number.

1. $(\exists x)(f(x) \wedge \neg a(x))$ Justification:

2. $(\forall x)(d(x) \rightarrow a(x))$ Justification:

3. $f(c) \wedge \neg a(c)$ Justification:

4. $d(c) \rightarrow a(c)$ Justification:

5. $\neg a(c) \wedge f(c)$ Justification: 3, $\wedge$ is commutative.

6. $\neg a(c)$ Justification:

7. $f(c)$ Justification:

8. $\neg d(c)$ Justification:

9. $f(c) \wedge \neg d(c)$ Justification:

10. $(\exists x)(f(x) \wedge \neg d(x))$ Justification: