
General directions: Read each problem carefully and do exactly what is requested. Show all your work neatly. Use complete sentences and use notation correctly. Make your arguments and proofs as complete as possible. Remember that what is illegible or incomprehensible is worthless. Show me all the magic on the page.

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1. (10 pts.) Construct the truth tables for each of the following compound propositions in the space provided:

(a) $\neg p \rightarrow q$

(b) $\neg(p \rightarrow q)$

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2. (5 pts.) Suppose p and q are propositions. What is the difference between "$p \leftrightarrow q$" and "$p \Leftrightarrow q$"?

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3. (10 pts.) Write the contrapositive and converse of the statement, "If horses have wings, then the Pegasus needs dramamine," and label these unambiguously. Which of these fails to be equivalent to the original statement?

4. (15 pts.) Let F(x,y) be the statement "x can fool y". The universe of discourse is all people. Use quantifiers to express each of the following statements:

(a) There is someone whom Frodo can fool.

(b) Gandalf fools everyone.

(c) The only person Frodo can fool is himself.

5. (5 pts.) Determine the truth value of each of the following statements if the universe of discourse of each variable is the set of natural numbers, $\mathbb{N} = \{0,1,2,...\}$.

(a) $(\forall x)(\exists y)(x + y = y)$

(b) $(\exists x)(\forall y)(x + y = y)$

6. (5 pts.) If A is a countable set and B is an uncountable set, must A - B be countable? Briefly explain.

7. (15 pts.) Suppose $A = \{\emptyset, \{\emptyset\}, 2\}$ and $B = \{2,3\}$. Then

$A \cup B =$

$A \times B =$

$|P(A)| =$

8. (5 pts.) Suppose A and B are subsets of a universal set U. Show that $A - B = \emptyset \rightarrow A \subseteq B$.
[Hints: (1) $x \in A \rightarrow x \notin \emptyset \rightarrow$ ... .
(2) $A - B = \emptyset \rightarrow \forall x(x \in A - B \rightarrow x \in \emptyset)$. The contrapositive of the implication within the parentheses here is useful in dealing with the ellipsis in hint #1.]

9. (5 pts.) If $f:X \rightarrow Y$ is a function, f^{-1} may be used to denote two quite different things. What are they? [Use complete sentences.]

10. (15 pts.) Suppose that $f:\mathbb{R} \to \mathbb{Z}$ is the function defined by the formula $f(x)= \lfloor x \rfloor$, and suppose that $A = \{x \in \mathbb{R} \mid -3 \leq x \leq 3\}$ and $B = \{x \in \mathbb{R} \mid -1 < x \leq \pi\}$. Using appropriate notation, give each of the following.

$A - B =$

$f(B) =$

$f^{-1}(\{1,3\}) =$

11. (5 pts.) What is the value of the following sum of terms of a geometric progression? [Hint: You may wish to re-index the varmint.]

$$\sum_{j=1}^{8} 2^j =$$

12. (5 pts.) Suppose $g:A \to B$ and $f:B \to C$ are functions. Prove exactly one of the following propositions. Indicate clearly which you are demonstrating.

(a) If $f \circ g:A \to C$ is injective, then g is injective.

(b) If $f \circ g:A \to C$ is surjective, then f is surjective.