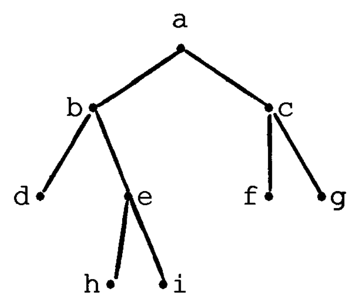
General directions: Read each problem carefully and do exactly what is requested. Show all your work neatly. Use complete sentences and use notation correctly. Make your arguments and proofs as complete as possible. Remember that what is illegible or incomprehensible is worthless.

1. (15 pts.) Given the ordered rooted tree below, list the vertices in the order in which they are visited in (a) a preorder traversal of the tree, (b) an inorder traversal of the tree, and (c) a postorder traversal of the tree. Place your lists in the appropriate place below.



(a) preorder traversal :

(b) inorder traversal :

(c) postorder traversal :

2. (10 pts.) (a) Using the rooted binary tree in Problem 1 to obtain appropriate prefix codes, produce the string of zeros and ones that encodes the word **dig**.

dig :

(b) Construct the ordered rooted binary tree representing the following algebraic identity: ((A + B)/C) = ((A/C) + (B/C))

3. (15 pts.) Suppose R and S are relations on the set A = {a,b,c} represented by the matrices given below.

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{M}_S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) If we assume that the matrices were constructed using the order listed above for the set A, then

S =

(b) What is the matrix representing $R \cup S$?

$\mathbf{M}_{R \cup S}$ =

(c) Construct the matrix representing the relation $T = R^{-1}$ that is the inverse of R.

$\mathbf{M}_T$ =

4. (10 pts.) (a) What is the numerical value of the postfix expression below?

3 2 * 2 ↑ 5 3 - 8 4 / * - =

(b) What is the numerical value of the prefix expression below?

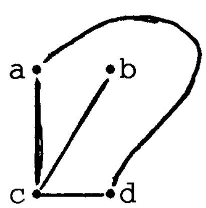
+ - * 2 3 5 / ↑ 2 3 4 =

5. (15 pts.) (a) Draw a directed graph G_1 whose adjacency matrix is given on the left below.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(b) Now draw the underlying undirected graph G_2 for the directed graph G_1 of part (a) of this problem.

(c) Is $G_2 = (V_2,E_2)$, above, isomorphic to the simple graph $G_3 = (V_3,E_3)$ given below? Either display an isomorphism $f:V_2 \to V_3$ or very briefly explain why there is no such function by revealing an invariant that one graph has that the other doesn't.



6. (10 pts.) Recall that the composition of two relations R and S on a set A is given by

$S \circ R = \{ (a,c) \in A \times A \mid (\exists b)(b \in A \text{ and } (a,b) \in R \text{ and } (b,c) \in S)\}.$

Also, recall that the n^{th} composition power of a relation on a set is defined recursively by $R^1 = R$, and for each $n \in \mathbb{N}$, if $n \geq 1$, then $R^{n+1} = R^n \circ R$.

Prove that if R is a reflexive relation on a nonempty set A, then $R \subseteq R^n$ for every positive integer n.

7. (15 pts.) (a) How many vertices does a tree with 37 edges have?

(b) What is the maximum number of leaves that a binary tree of height 6 can have?

(c) If a full 3-ary tree has 24 internal vertices, how many leaves does it have?

8. (10 pts.) Suppose that A is the set consisting of all real-valued functions with domain consisting of the interval [-1,1] . Let R be the relation on the set A defined as follows:

$$R = \{ (f,g) \mid (\exists C)(C \in \mathbb{R} \text{ and } f(0) - g(0) = C) \}$$

Prove that R is an equivalence relation on the set A.