Read Me First: *Show all essential work neatly. Use correct notation when presenting your computations. Write using complete sentences. In particular, be very careful when using "=", **equals**, and "⇒", **implies**. Do not "box" your answers. Communicate.*

1. (6 pts.) Find the amplitude, period, and phase shift of the following function: $y = -2\sin(2\pi x + (\pi/2))$
$= -2\sin(\ 2\pi(x - (-1/4))\ )$

Amplitude = 2; Period = $2\pi/2\pi = 1$; Phase Shift = $-1/4$ or $1/4$ to the left.

2. (6 pts.) Write the equation of a sine function that has all the given characteristics:

Amplitude = 2π Period = $1/3$ Phase Shift: $-(1/6)$

$$y = 2\pi\sin(6\pi(x - (-1/6))) = 2\pi\sin(6\pi x + \pi)$$

3. (10 pts.) Carefully sketch $y = 3\sin(2x - \pi)$ through one period. You will need the amplitude, period, and phase shift to do this properly. Label very carefully. **NOTE:** *This graph is provided in a separate document.*

4. (18 pts.) Fill in the following table with the information requested concerning domain, range, and period.

Function Name	Domain (in radians)	Range	Period (in radians)
$\sin(\theta)$	$\mathbb{R}$	$[-1,1]$	2π
$\cos(\theta)$	$\mathbb{R}$	$[-1,1]$	2π
$\tan(\theta)$	A, below	$\mathbb{R}$	π
$\cot(\theta)$	B, below	$\mathbb{R}$	π
$\sec(\theta)$	A, below	$(-\infty,-1]\cup[1,\infty)$	2π
$\csc(\theta)$	B, below	$(-\infty,-1]\cup[1,\infty)$	2π

$A = \{\ x \in \mathbb{R} : x \neq (2k + 1)(\pi/2),\ k \text{ any integer}\ \}$

$B = \{\ x \in \mathbb{R} : x \neq k\pi,\ k \text{ any integer}\ \}$

5. (5 pts.) Establish the following identity. Show all steps very, very carefully.

$$\frac{1 - \sin(\alpha)}{\cos(\alpha)} = \frac{\cos(\alpha)}{1 + \sin(\alpha)}$$

Proof:

$$\frac{1 - \sin(\alpha)}{\cos(\alpha)} = \frac{[1 - \sin(\alpha)]\cdot[1 + \sin(\alpha)]}{\cos(\alpha)[1 + \sin(\alpha)]}$$

$$= \frac{1 - \sin^2(\alpha)}{\cos(\alpha)[1 + \sin(\alpha)]}$$

$$= \frac{\cos^2(\alpha)}{\cos(\alpha)[1 + \sin(\alpha)]}$$

$$= \frac{\cos(\alpha)}{1 + \sin(\alpha)} \qquad //$$

6. (5 pts.) Obtain the exact value of $\cos(\pi/8)$. **Show clearly and neatly all the uses of appropriate identities.**

$\cos(\pi/8) = ([1 + \cos(2(\pi/8))]/2)^{1/2} = ([1 + \cos((\pi/4))]/2)^{1/2}$

$= ([1 + (2^{1/2}/2)]/2)^{1/2} = [(2 + 2^{1/2})^{1/2}]/2$

7. (5 pts.) If $\csc(\theta) = 4$ and $\cos(\theta) < 0$, what is the exact value of $\sin(2\theta)$?? **Show clearly and neatly all your uses of appropriate identities.**
$\sin(\theta) = 1/\csc(\theta) = 1/4$ and $\cos(\theta) = -(1 - \sin^2(\theta))^{1/2} = -15^{1/2}/4$ since $\cos(\theta) < 0$. So

$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2\cdot(1/4)\cdot(-15^{1/2}/4) = -15^{1/2}/8$

8. (5 pts.) Express the following product as a sum containing only sines or cosines.

$\sin(5\theta)\cos(3\theta) = (1/2)[\sin(8\theta) + \sin(2\theta)]$

9. (10 pts.) Find the exact value of each of the following expressions if $\tan(\alpha) = -5/12$ with $\pi/2 < \alpha < \pi$ and $\sin(\beta) = -1/2$ with $\pi < \beta < 3\pi/2$. **Show all your uses of appropriate identities.**

$\sin(\alpha) = 5/13$ and $\cos(a) = -12/13$ since $\pi/2 < \alpha < \pi$.

$\cos(\beta) = -3^{1/2}/2$ since $\pi < \beta < 3\pi/2$.

$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta) = -(5\cdot 3^{1/2} + 12)/26$

$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) = (12\cdot 3^{1/2} + 5)/26$

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10. (20 pts.) Time to pay the piper.... Give the exact values for the following:

(a) $\sin(0) = 0$

(b) $\sin(\pi/6) = 1/2$

(c) $\sin(\pi/4) = 2^{1/2}/2$

(d) $\sin(\pi/3) = 3^{1/2}/2$

(e) $\sin(\pi/2) = 1$

(f) $\cos(0) = 1$ [You could use the Complementary Angle Theorem to get the cosines, folks. LOOK !]

(g) $\cos(\pi/6) = 3^{1/2}/2$

(h) $\cos(\pi/4) = 2^{1/2}/2$

(i) $\cos(\pi/3) = 1/2$

(j) $\cos(\pi/2) = 0$

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11. (10 pts.) In order to get a neat identity for $\cos(\alpha) + \cos(\beta)$, one begins with the identity

(*) $\cos(x + y) + \cos(x - y) = 2\cdot\cos(x)\cos(y)$

and sets $x + y = \alpha$ and $x - y = \beta$ in the left side of the identity. To make the substitution uniform, it is necessary to replace the "x" and "y" on the right side of (*) with what they are in terms of "α" and "β" in the system of linear equations

$$x + y = \alpha$$

$$x - y = \beta.$$

Solve for x and y in this system.

$x = (\alpha + \beta)/2$ and $y = (\alpha - \beta)/2$ quickly, easily.