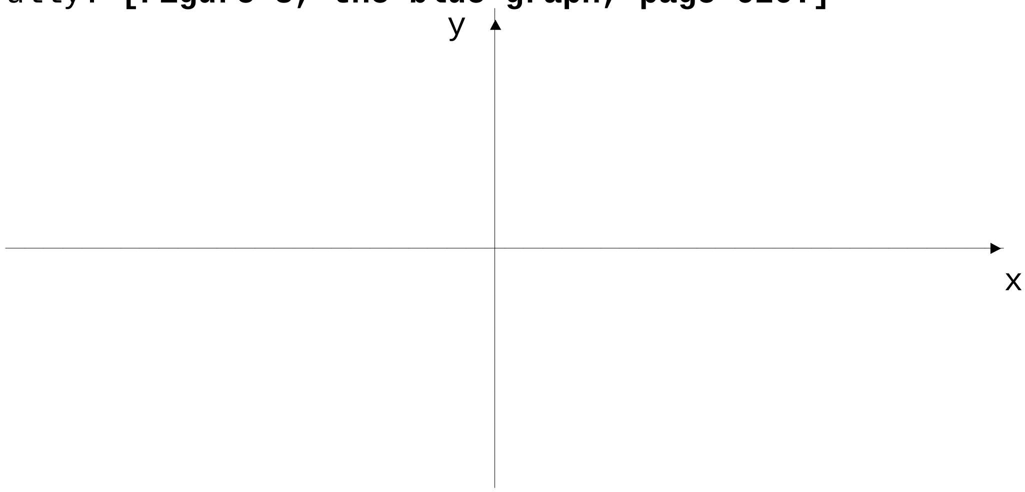
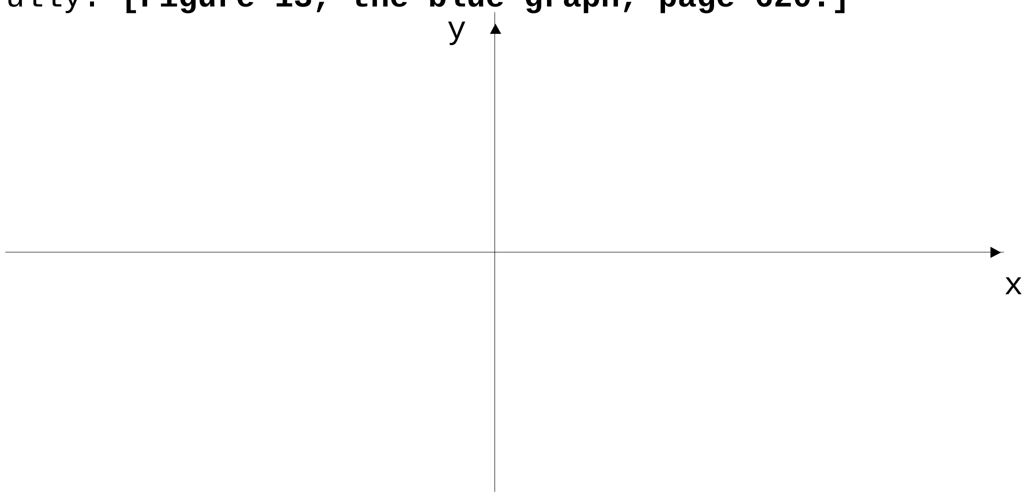
**Read Me First:** *Show all essential work neatly. Use correct notation when presenting your computations. Write using complete sentences. In particular, be very careful when using "=",* ***equals****, and "⇒",* ***implies****. Do not "box" your answers. Communicate.*

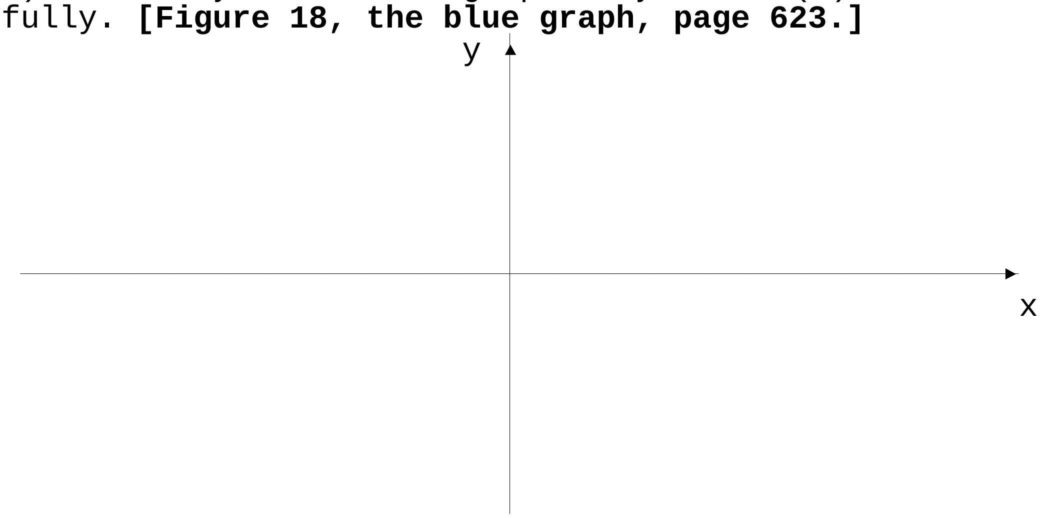
1. (5 pts.) Carefully sketch the graph of $y = \sin^{-1}(x)$. Label very carefully. **[Figure 8, the blue graph, page 616.]**




2. (5 pts.) Carefully sketch the graph of $y = \cos^{-1}(x)$. Label very carefully. **[Figure 13, the blue graph, page 620.]**




3. (5 pts.) Carefully sketch the graph of $y = \tan^{-1}(x)$. Label very carefully. **[Figure 18, the blue graph, page 623.]**



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4. (5 pts.) Use your calculator to find the value of $\sec^{-1}(-4/3)$ rounded to two decimal places.

$\sec^{-1}(-4/3) \approx 2.42$

Here's why: $\sec^{-1}(-4/3) = y$ if, and only if $\sec(y) = -4/3$ and $\pi/2 < y \le \pi$. This last clause is equivalent to $\cos(y) = -3/4$ and $\pi/2 < y \le \pi$. Now use your calculator to obtain $\cos^{-1}(-3/4)$ *in radian mode*. A near miss: 138.59°.

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5. (5 pts.) Find the exact value of $\sin^{-1}(\sin(-7\pi/6))$.

$\sin^{-1}(\sin(-7\pi/6)) = \sin^{-1}(\sin(\pi/6)) = \pi/6$.

The problem is that $-7\pi/6$ is outside of the interval $[-\pi/2, \pi/2]$. Get the reference angle and then take care of the issue of sign to find the $\theta$ in $[-\pi/2, \pi/2]$ where $\sin(\theta) = \sin(-7\pi/6)$. Then use $\sin^{-1}(\sin(\theta)) = \theta$. [This is the most general approach. There are other ways to do this particular problem because $\pi/6$ is one of the nice guys.]

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6. (5 pts.) Write $\sin(2\sin^{-1}(v))$ as an algebraic expression containing v.

Again, we must use $\sin^{-1}(\sin(\theta)) = \theta$. In addition, we must put some of our wonderful identities to work. Here, we need both $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ and the beloved pythagorean varmint $\sin^2(\theta) + \cos^2(\theta) = 1$.

$\sin(2\sin^{-1}(v)) = 2\sin(\sin^{-1}(v))\cos(\sin^{-1}(v))$ //Let $\theta = \sin^{-1}(v)$ in in the double angle thingy.//

$= 2v(1-v^2)^{1/2}$

//Set $\theta = \sin^{-1}(v)$ in the pythagorean beast and solve for $\cos(\sin^{-1}(v))$, all the while keeping in mind that cosine is non-negative in the interval $[-\pi/2, \pi/2]$, since we have $-\pi/2 \le \sin^{-1}(v) \le \pi/2$ provided $-1 \le v \le 1$.//

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7. (5 pts.) Find the exact value of $\tan(2\cdot\tan^{-1}(3/4))$.

$\tan(2\cdot\tan^{-1}(3/4)) = 2\tan(\tan^{-1}(3/4))/[1 - \tan^2(\tan^{-1}(3/4))]$

$= (6/4)/[1 - (9/16)] = 24/7$.

...same theme, another variation...

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8. (5 pts.)(a) Obtain all solutions to the equation below, and then (b) list the solutions $\theta$ with $0 \le \theta < 2\pi$.

$$\sin(3\theta) = -1$$

(a) The given equation is equivalent to $3\theta = 3\pi/2 + 2k\pi$, for k any integer. Solving this for $\theta$ yields $\theta = \pi/2 + (2\pi/3)k$ for k any integer.

(b) The solutions in the desired interval are $\pi/2$, $7\pi/6$, $11\pi/6$.

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9. (10 pts.) A right triangle has one angle of 35° and one leg of length 100 meters. What are the two possible lengths for the hypotenuse?? [You may want to sketch the two situations.]

Depending on whether the leg is adjacent or opposite the angle, we have either $\cos(35°) = 100/h$ or $\sin(35°) = 100/h$. Thus, either $h = 100/\cos(35°) \approx 122.08$ meters or $h = 100/\sin(35°) \approx 174.34$ meters.

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10. (5 pts.) A triangle has two sides with lengths 5 feet and 8 feet. If the two sides meet in an angle of 30°, what is the exact length of the third side??

If x denotes the length of the third side, then from the Law of Cosines, we have $x^2 = 8^2 + 5^2 - 2(8)(5)(\cos(30°) = 89 - 40\cdot 3^{1/2}$. Thus, $x = (89 - 40\cdot 3^{1/2})^{1/2}$, exactly. Note that $x \approx 4.44$ feet.

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11. (10 pts.) Use the Law of Sines to solve the triangle with $\alpha = 110°$, $\gamma = 30°$, and $c = 6$. You may assume that the standard labelling scheme is used.

The missing pieces are $\beta = 40°$, $a = 12\sin(110°) \approx 11.28$, and $b = 12\sin(40°) \approx 7.71$.

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12. (5 pts.) Determine whether one, two, or no triangles result from the following data. You do not have to solve the triangles that might result. You may assume that the standard labelling scheme is used.

$$a = 3,\ b = 6,\ \alpha = 32°$$

If there were such a triangle, we would have to have $\sin(32°)/3 = \sin(\beta)/6$ true for some $\beta$. Unfortunately, this last equation is equivalent to $\sin(\beta) = 2\cdot\sin(32°) > 2\cdot\sin(30°) = 1$. There is no solution to this, and thus, no triangle.

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13. (10 pts.) To measure the height of the top of a distant object on a level plane, a surveyor takes two sightings of the top of the object 1000 feet apart. The first sighting, which is nearest the object, results in an angle of elevation of 60°. The second sighting, which is most distant from the object, results in an angle of elevation of 30°. If the transit used to make the sightings is 5 feet tall, what is the height of the object. [Hint: Make a diagram of the situation. The distance from the base of the object is unknown. ]

Let h denote the height of the object, x the length of the side opposite the 60° sighting, and d the length of the side adjacent to the 60° angle. The sides of length d and x meet in a right angle. Then we have $h = x + 5$, and the system of equations $\tan(60°) = x/d$ and $\tan(30°) = x/(d + 1000)$. Solving for x yields $x = 1000(\tan(30°))/(1 - \cot(60°)\tan(30°))$. Alternatively, you could recognize that, here, the hypotenuse of the triangle created by the 60° sighting is one of the two equal sides of the isosceles 30°,30°,120° triangle formed from the 30° sighting. Thus, that hypotenuse has length 1000 feet. This leads to the very simple equation $x = 1000\cdot\sin(60°)$. In either case, you will get $x \approx 866.03$ feet and so $h \approx 871.03$ feet. **[Warning: The cute isosceles triangle thingy doesn't usually happen. The other approach is more general.]**

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14. (5 pts.) If the polar coordinates of a point are given by $(r,\theta) = (9.5,110°)$, find the rectangular coordinates for the point. In doing this, make clear which values are exact and which are approximations.

$(x,y) = ((9.5)\cos(110°),(9.5)\sin(110°)) \approx (-3.25, 8.93)$

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15. (5 pts.) If the rectangular coordinates of a point are given by $(x,y) = (-5,-5\sqrt{3})$, obtain polar coordinates for the point.

It's easy to see $r^2 = 100$. Thus, use $r = 10$, to keep things simple. It turns out that the reference angle $\theta_r$ satisfies the equation $\tan(\theta_r) = 3^{1/2}$. Thus, because the point lies in the third quadrant, we may use either $\theta = 240°$ or $\theta = 4\pi/3$. From here, it is easy to list all pairs $(r,\theta)$ that represent the point.

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16. (10 pts.) (a) Obtain all solutions to the equation below, and then (b) list the solutions $\theta$ with $0 \le \theta < 2\pi$.

$$2\cdot\sin^2(\theta) + 3\cdot\sin(\theta) + 1 = 0$$

(a) The given equation is equivalent to

$$(2\sin(\theta) + 1)(\sin(\theta) + 1) = 0,$$

which is equivalent to $\sin(\theta) = -1/2$ or $\sin(\theta) = -1$. All solutions to $\sin(\theta) = -1/2$ are given by $\theta = 7\pi/6 + 2k\pi$ or $\theta = 11\pi/6 + 2k\pi$, k any integer, and all solutions to $\sin(\theta) = -1$ are given by $\theta = 3\pi/2 + 2k\pi$, k any integer. (b) The solutions in the desired interval are $7\pi/6$, $3\pi/2$, and $11\pi/6$.