Read Me First: *Show all essential work neatly. Use correct notation when presenting your computations. Write using complete sentences. In particular, be very careful when using "=",* ***equals****, and "$\Rightarrow$",* ***implies****. Do not "box" your answers. Communicate.*

1. (15 pts.) Identify each of the following polar equations as completely as possible by transforming each equation to rectangular coordinates. [These are fairly easy!!]

(a) $\theta = -(1/6)\pi$
This is the straight line through the origin defined by the equation $y = -\tan(\pi/6)\cdot x = -3^{-1/2}\cdot x$.

(b) $2r\cdot\sin(\theta) = 3r\cdot\cos(\theta)$
This is the straight line through the origin defined by the equation $2y = 3x$.

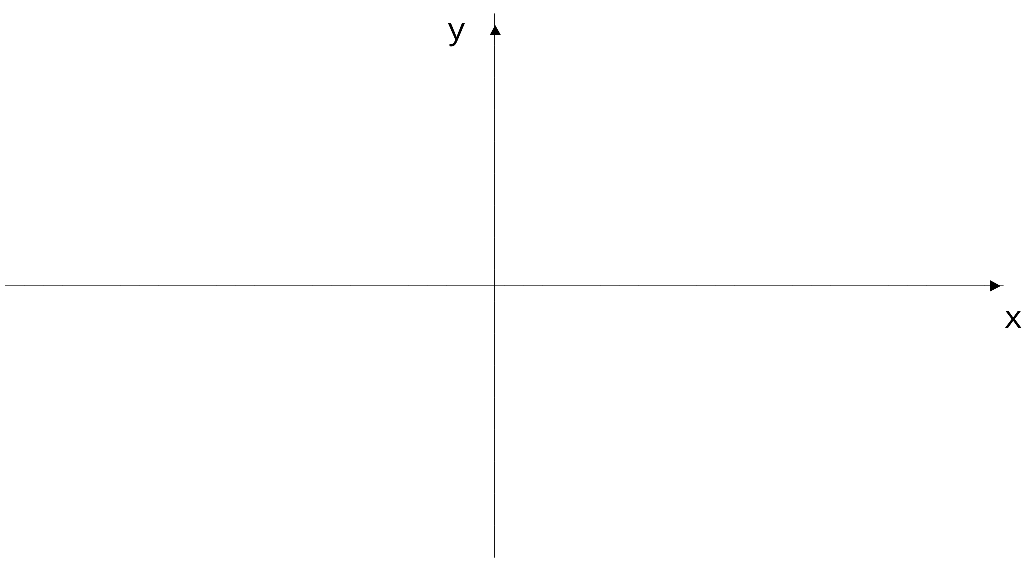
(c) $r = 3$
This is the circle centered at the origin with radius equal to 3. Its rectangular equation is $x^2 + y^2 = 9$.

(d) $r = 2\cdot\sin(\theta)$

This is the circle with the following rectangular equation: $x^2 + (y - 1)^2 = 1$. Consequently, its center is at (0,1) and it has a radius equal to 1.

(e) $r\cdot\cos(\theta) = -4$
This is simply the line parallel to the x-axis defined by $x = -4$.

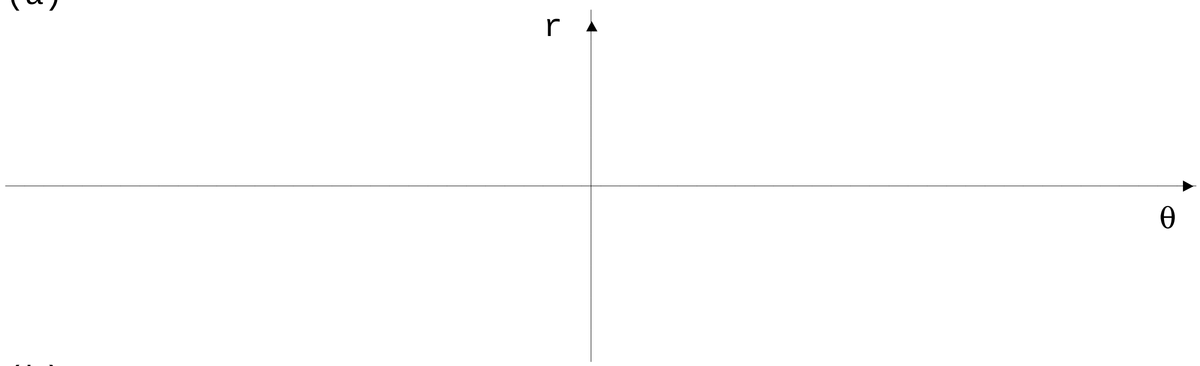
2. (10 pts.) Very carefully sketch the graph of the equation $x^2 = -4y$ below. [**See tr-t4g2.pdf.**]



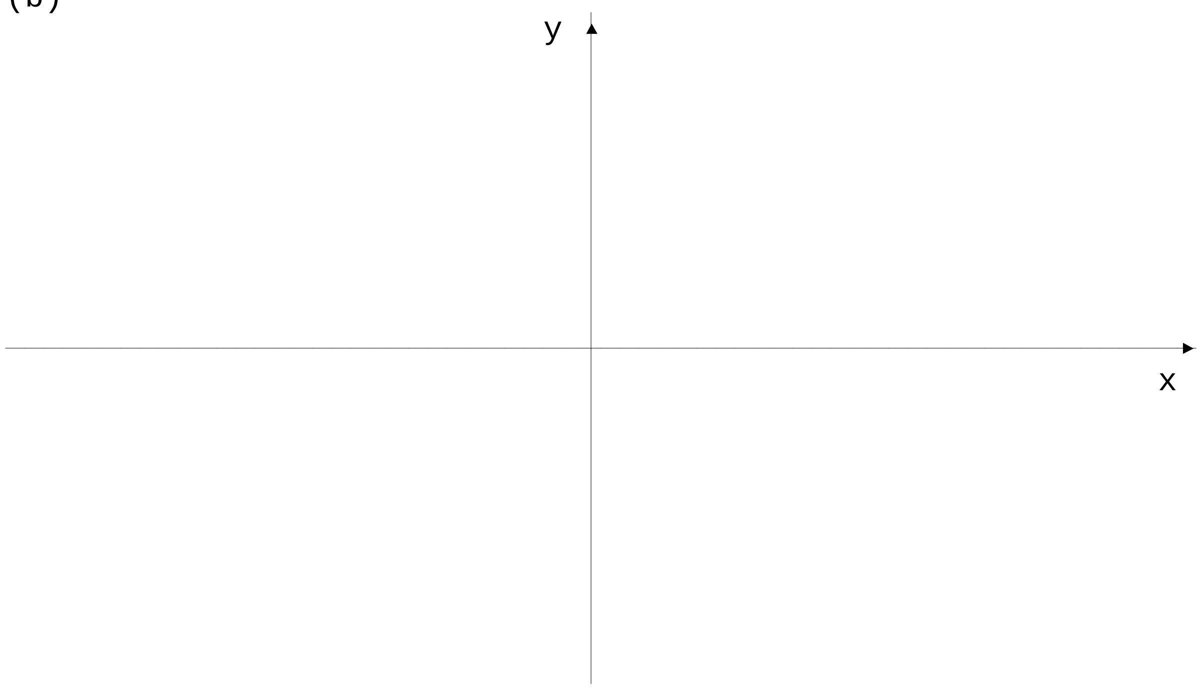
3. (15 pts.) Sketch the given curve in polar coordinates. Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph on the coordinate system provided. (b) Then translate this graph to the polar one. [**See tr-t4g3.pdf.**]

Equation: $r = 2\cdot\cos(2\theta)$

(a)



(b)



4. (10 pts.) Write each expression in the standard form a + bi.

(a) $(8 - 5i) + (-9 + 2i) = -1 - 3i$

(b) $13/(4 - 3i) = (52/25) + (39/25)i$

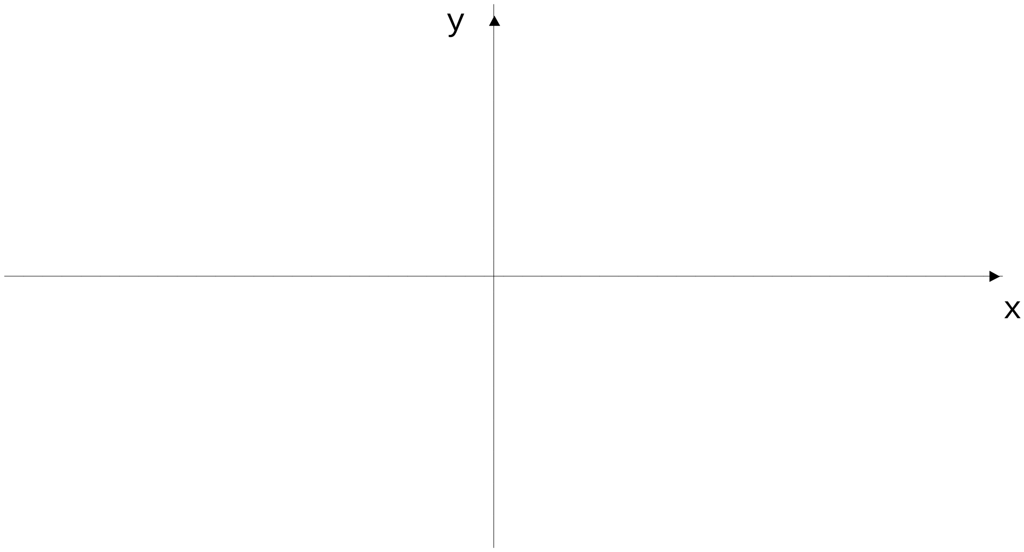
(c) $4i^5 - 6i^7 = 4i - (6)(-i) = 10i$

(d) $(2 - 4i)\cdot(-3 + 5i) = 14 + 22i$

(e) $[2(\cos 30° + i\cdot\sin 30°)]^5 = 2^5[\cos(150°) + i\cdot\sin(150°)]$

$= -16\cdot 3^{1/2} + 16i$

5. (10 pts.) Very carefully sketch the graph of the equation $(1/9)x^2 + (1/4)y^2 = 1$ below. [**See tr-t4g5.pdf.**]



6. (5 pts.) Solve the following equation in the complex number system: $x^4 - 4\cdot x^2 - 5 = 0$

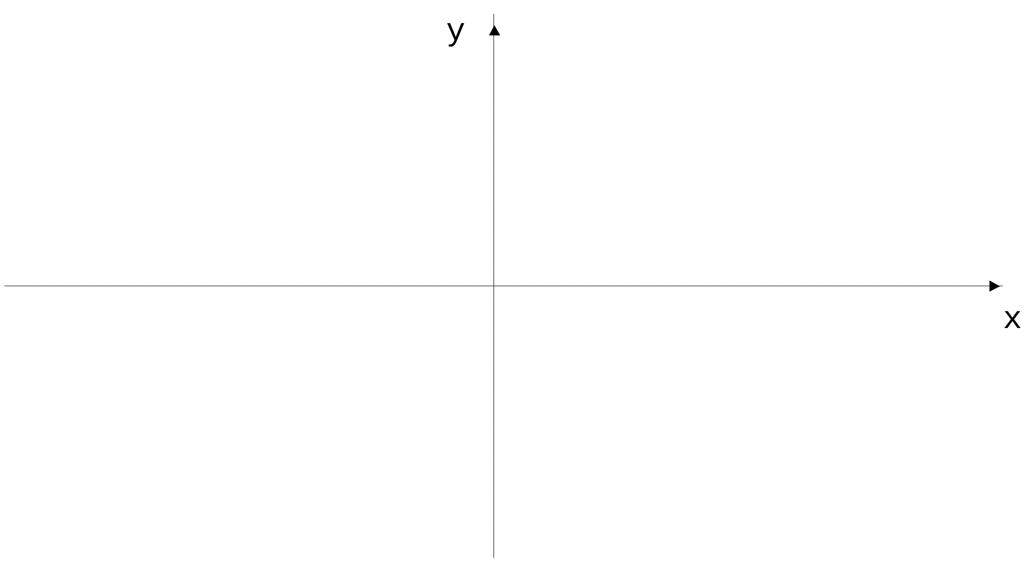
By factoring, we have

$$x^4 - 4\cdot x^2 - 5 = 0 \quad \Leftrightarrow \quad (x^2 - 5)(x^2 + 1) = 0$$

$$\Leftrightarrow \quad (x - 5^{1/2})(x + 5^{1/2})(x + i)(x - i) = 0$$

Thus, $x = 5^{1/2}$ or $x = -5^{1/2}$ or $x = -i$ or $x = i$. If necessary, you can use the quadratic formula to get the roots of $x^2 - 5 = 0$ and $x^2 + 1 = 0$. Keep firmly in mind, however, the quadratic formula provides a factorization of quadratics.

7. (10 pts.) Very carefully sketch the graph of the equation $y^2 - x^2 = 1$ below. [**See tr-t4g7.pdf.**]



8. (10 pts.) Find all the complex cube roots of $3^{1/2} + i$. Leave your answer in polar form with the arguments given in degrees.

If $z = 3^{1/2} + i$, then we can write z in polar form as $z = 2[\cos(30°) + \sin(30°)i]$. The three cube roots are

$$w_0 = 2^{1/3}[\cos(10°) + \sin(10°)i],$$

$$w_1 = 2^{1/3}[\cos(130°) + \sin(130°)i], \text{ and}$$

$$w_2 = 2^{1/3}[\cos(250°) + \sin(250°)i]$$

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9. (5 pts.) Find the vertex, focus, and directrix of the parabola that has the equation given below.

$$y^2 - 4y = x + 4.$$

By performing the usual algegraic magic you can transform the equation above into the standard form equation

$$(y - 2)^2 = 4(1/4)(x - (-8)).$$

Using this, you can easily see that the vertex is (-8,2), the focus is (-8 + (1/4), 2) = (-31/4, 2), and the directrix is the line defined by x = -8 - (1/4) = -33/4.

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10. (5 pts.) Find the center, foci, and vertices of the ellipse that has the equation given below.

$$4x^2 + y^2 + 4y = 0.$$

Again, by playing the complete-the-square game carefully, you should obtain the standard form equation

$$(x - 0)^2 + (1/4)(y - (-2))^2 = 1.$$

From this you should have $c = (4 - 1)^{1/2}$. Clearly the center is (0,-2), the two vertices are (0,0) and (0,-4), and the two foci are $(0,2 + 3^{1/2})$ and $(0,2 - 3^{1/2})$.

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11. (5 pts.) Find the center, foci, and vertices of the hyperbola that has the equation given below.

$$y^2 - 4x^2 -16x - 2y - 19 = 0$$

Finally, by being very puntilious in your algebraic prestidigitation, you can obtain the standard form equation

$$(1/4)(y - 1)^2 - (x - (-2))^2 = 1.$$

The center is (-2,1), the two vertices are (-2,3) and (-2,-1), and since $c = (4 + 1)^{1/2}$, the two foci are $(-2, 1 + 5^{1/2})$ and $(-2, 1 - 5^{1/2})$.