Student Number: Exam Number:

Read Me First: *Show all essential work neatly. Use correct notation when presenting your computations. Write using complete sentences. In particular, be very careful when using "=", **equals**, and "⇨", **implies**. Do not "box" your answers. Communicate.*

1. (18 pts.) Fill in the following table with the information requested concerning domain, range, and period.

Function Name	Domain (in radians)	Range	Period (in radians)
sin(θ)			
cos(θ)			
tan(θ)			
cot(θ)			
sec(θ)			
csc(θ)			

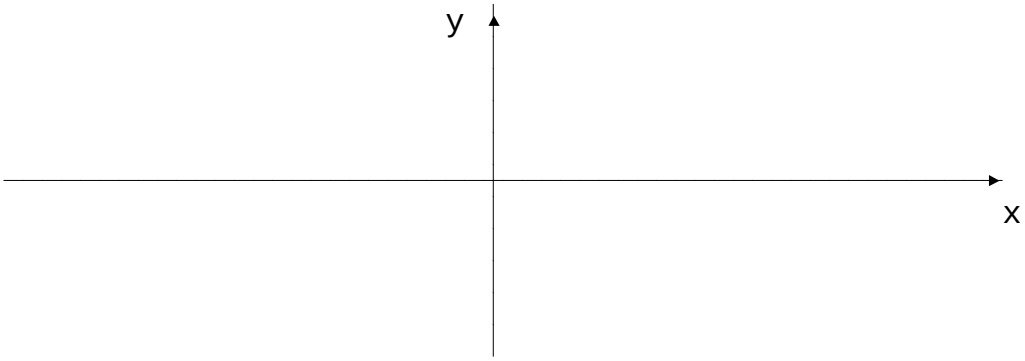
2. (7 pts.) If the point (2,-5) is on the terminal side of an angle θ, obtain the exact value of each of the six trigonometric functions of θ.

sin(θ) = cos(θ) =

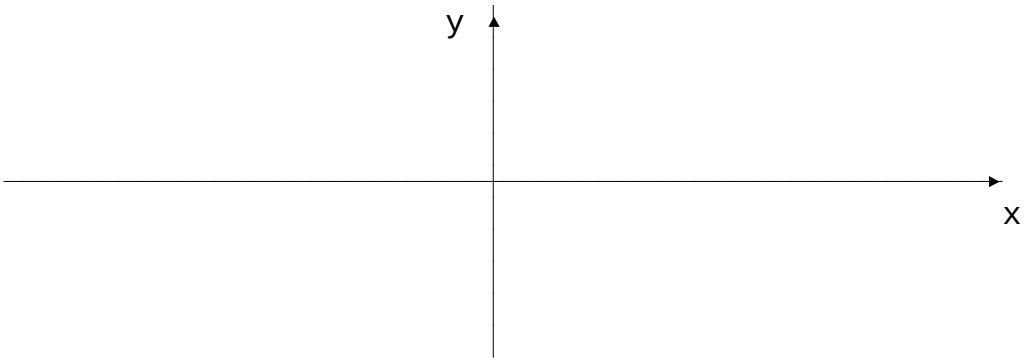
tan(θ) = cot(θ) =

sec(θ) = csc(θ) =

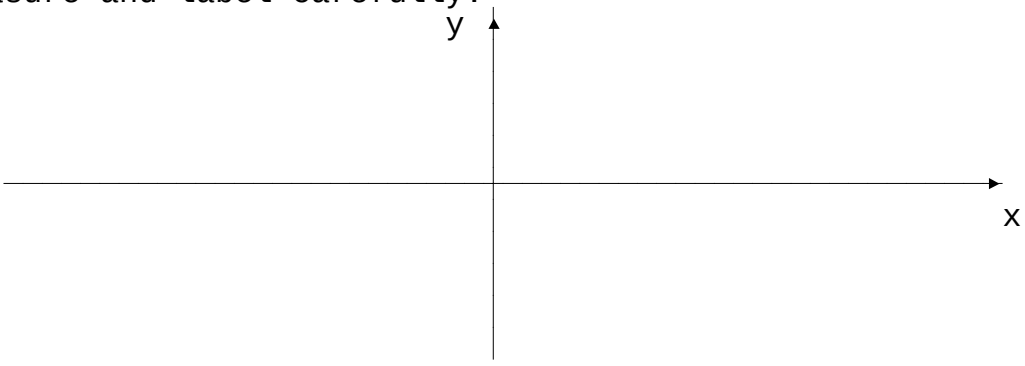
3. (6 pts.) Carefully sketch y = sin(x) through two periods that are symmetric about the origin. Use radian measure and label carefully.



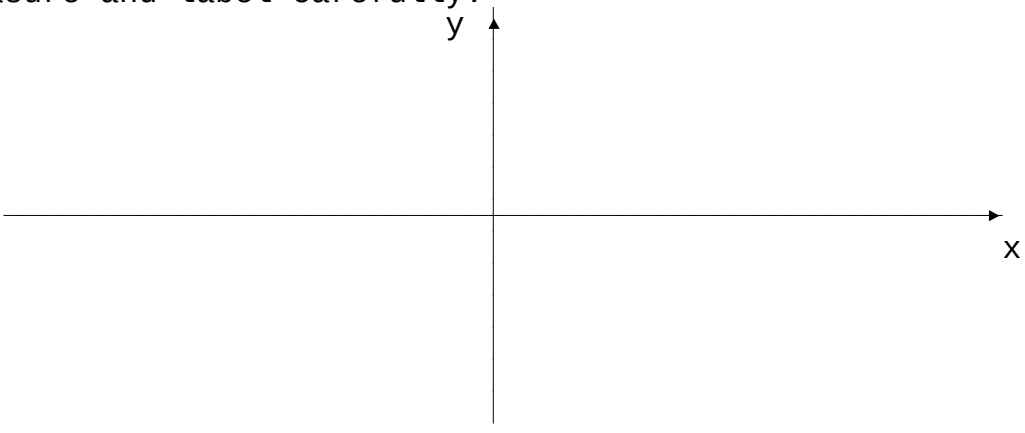
4. (6 pts.) Carefully sketch y = cos(x) through two periods that are symmetric about the origin. Use radian measure and label carefully.



5. (6 pts.) Carefully sketch y = tan(x) through two periods. Use radian measure and label carefully.



6. (7 pts.) Carefully sketch y = sec(x) through two periods. Use radian measure and label carefully.



7. (5 pts.) Suppose $\sec\theta = 4$ and $\tan\theta < 0$. What is the exact value of each of the remaining trigonometric functions?

$\cos(\theta) =$ $\tan(\theta) =$

$\sin(\theta) =$ $\csc(\theta) =$

$\cot(\theta) =$

8. (4 pts.) If $\theta = -240°$, what is the radian measure of θ as an exact multiple of π??

$\theta =$

9. (4 pts.) If $\theta = 7\pi/6$ in radian measure, what is the value of θ in degrees??

$\theta =$

10. (4 pts.) If $s = 5$ meters is the length of an arc of a circle of radius $r = 3$ meters subtended by a central angle θ, what is the exact value of θ in degrees??

$\theta =$

11. (4 pts.) If $\theta = 61°50'23''$, convert θ to a decimal in degrees rounded to two decimal places.

$\theta =$

12. (4 pts.) If $\theta = 30.421°$, convert θ to D°M′S″ form with the answer rounded to the nearest second.

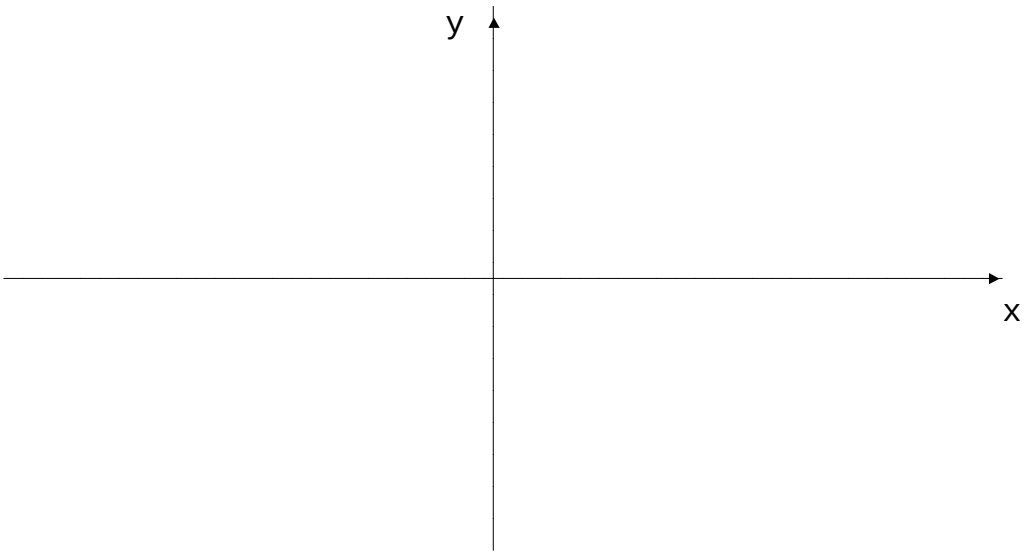
$\theta =$

13. (6 pts.) Using DeMoivre's Theorem, find all the complex fourth roots of i. [Don't confuse fourth roots with fourth powers!!]

14. (6 pts.) Write the **equation** of a sine function that has all the given characteristics:

Amplitude = 4π Period = 3 Phase Shift: -(3/4)

15. (6 pts.) Carefully sketch $y = -4\cos(2x + \pi)$ through one period. You will need the amplitude, period, and phase shift to do this. *Label very carefully.*



16. (7 pts) Obtain the exact value of $\sin(9\pi/8)$. **Show all the uses of appropriate identities and formulas.** *[Hints: Reference angle? Quadrant??]*

$\sin(9\pi/8) =$

17. (5 pts.) Find the exact value of $\sin^{-1}(\sin(-7\pi/5))$.

$\sin^{-1}(\sin(-7\pi/5)) =$

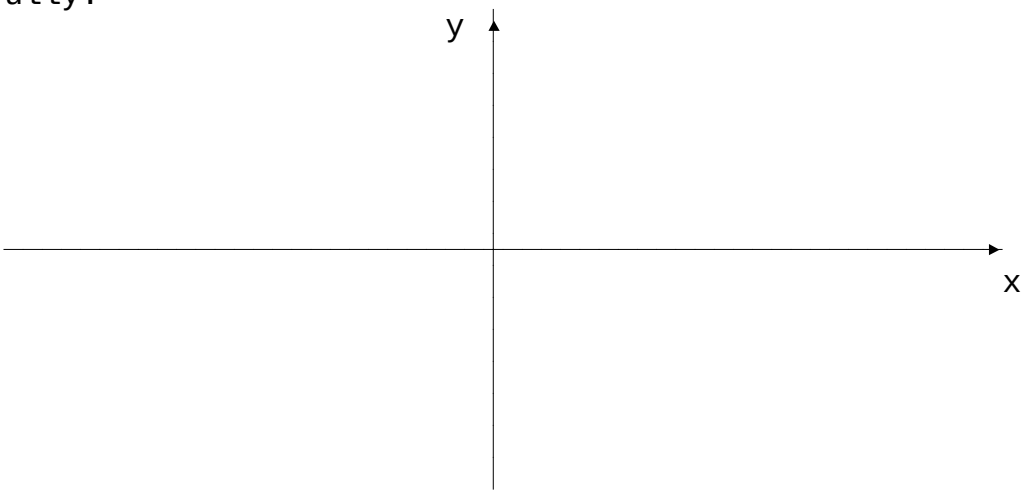
18. (5 pts.) Write $\sin(\cos^{-1}(u) + \sin^{-1}(v))$ as an algebraic expression containing u and v.

$\sin(\cos^{-1}(u) + \sin^{-1}(v)) =$

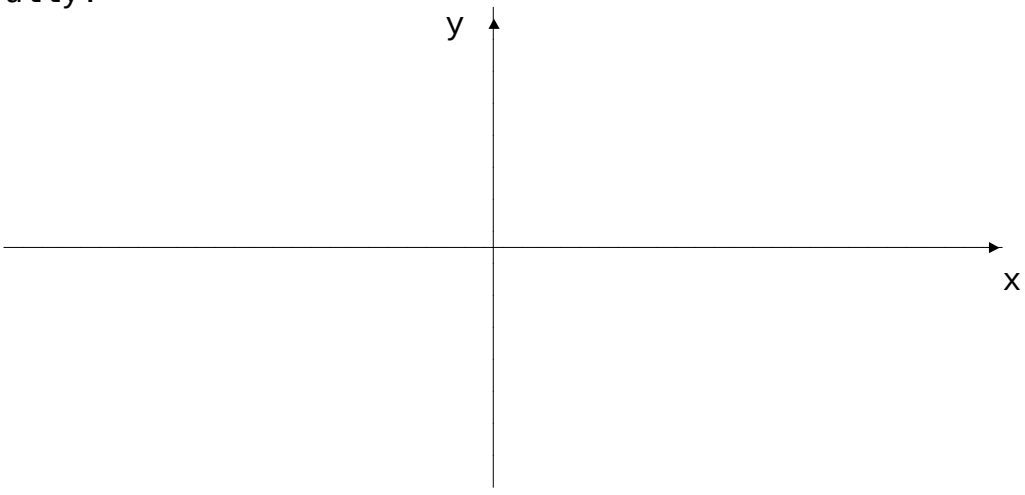
19. (5 pts.) Find the exact value of $\tan(2\cdot\tan^{-1}(3/4))$.

$\tan(2\cdot\tan^{-1}(3/4)) =$

20. (5 pts.) Carefully sketch the graph of $y = \cos^{-1}(x)$. Label very carefully.



21. (5 pts.) Carefully sketch the graph of $y = \tan^{-1}(x)$. Label very carefully.



22. (10 pts.) Establish the following identity.

$$\frac{1 - \cos(\alpha)}{\sin(\alpha)} = \frac{\sin(\alpha)}{1 + \cos(\alpha)}$$

23. (15 pts.) Very carefully complete the following derivation of a couple of half angle identities by giving the information requested and performing the computations needed.

(a) Write down the trigonometric identity giving cos(x + y) in terms of sums of products of sines and cosines of x's and y's.

cos(x + y) =

(b) Uniformly replace each instance of a y by x in the identity above to obtain an identity for cos(2x). Use exponentiation to clean up the products appearing.

cos(2x) = cos(x + x) =

(c) Using the Pythagorean identity connecting $\sin^2(x)$ and $\cos^2(x)$, replace $\sin^2(x)$ in the identity for cos(2x) and then solve for $\cos^2(x)$ in terms of what remains.

$\cos^2(x)$ =

(d) Uniformly replace x by $\theta/2$ in the identity for $\cos^2(x)$ that you obtained in part (c) above in order to obtain an identity for $\cos^2(\theta/2)$. Clean up the algebra.

$\cos^2(\theta/2)$ =

(e) Modify steps (c) and (d) appropriately to obtain a corresponding identity for $\sin^2(\theta/2)$. Show your work neatly.

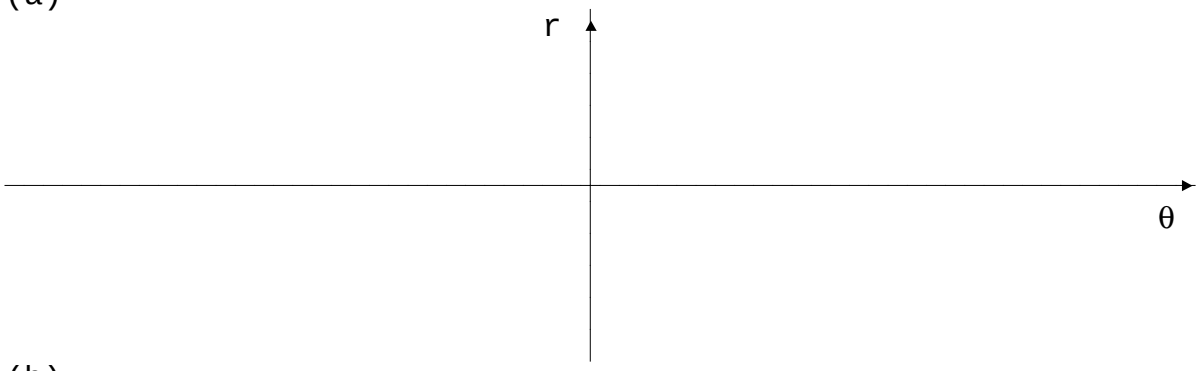
$\sin^2(\theta/2)$ =

24. (10 pts.) To measure the height of the top of a distant object , a surveyor takes two sightings of the top of the object 5000 feet apart. The first sighting, which is nearest the object, results in an angle of elevation of 45°. The second sighting, which is most distant from the object, results in an angle of elevation of 30°. If the transit used to make the sightings is 5 feet tall, what is the height of the object. You may assume the object is on a level plane with the base of the transit.

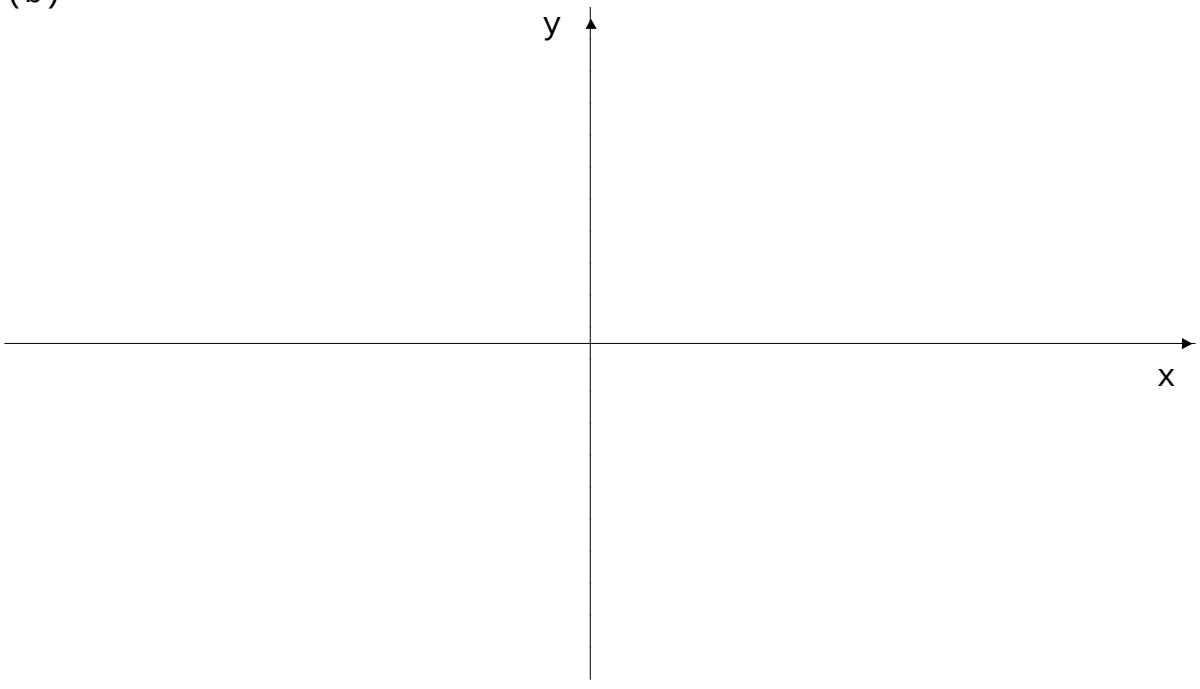
25. (15 pts.) Sketch the given curve in polar coordinates. Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph on the coordinate system provided. (b) Then translate this graph to the polar one.

Equation: $r = 1 + 2 \cdot \sin(\theta)$

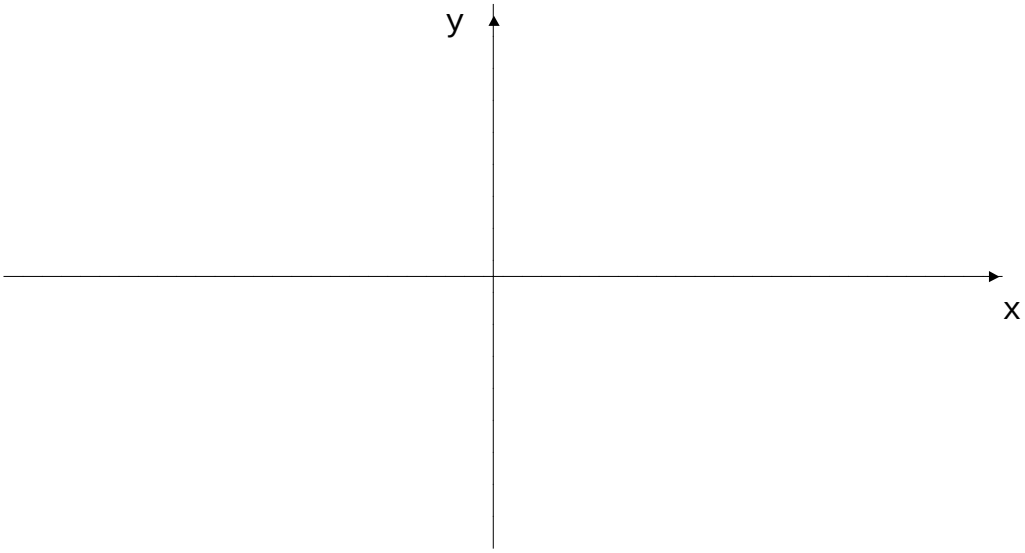
(a)



(b)



26. (10 pts.) Very carefully sketch the graph of the equation $(x + 1)^2 = -4(y - 2)$ below.



27. (5 pts.) Use the Law of Sines to solve the triangle with $\alpha = 115°$, $\gamma = 30°$, and $c = 3$. You may assume that the standard labelling scheme is used.

28. (10 pts.) Very carefully sketch the graph of the equation $(y + 1)^2 - (x + 2)^2 = 1$ below.