Problem 23 b from section 2-1:

Show that $\frac{1}{x^2+y^2}$ is an integrating factor for the equation

$$\left[y + x f(x^2+y^2)\right] dx + \left[y f(x^2+y^2) - x\right] dy = 0.$$

The equation with $\frac{1}{x^2+y^2}$ multiplied through is

$$\frac{y + x f(x^2+y^2)}{x^2+y^2} dx + \frac{y f(x^2+y^2) - x}{x^2+y^2} dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{(x^2+y^2)(1+2xy f'(x^2+y^2)) - 2y(y+xf(x^2+y^2))}{(x^2+y^2)^2}$$

$$= \frac{x^2 - y^2 + 2xy(x^2+y^2)f'(x^2+y^2) - 2xy f(x^2+y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x} = \frac{(x^2+y^2)(2xy f'(x^2+y^2) - 1) - 2x(y f(x^2+y^2) - x)}{(x^2+y^2)^2}$$

$$= \frac{x^2 - y^2 + 2xy(x^2+y^2) f'(x^2+y^2) - 2xy f(x^2+y^2)}{(x^2+y^2)^2}$$

$\therefore$ the new equation is exact.