# Chapter 15

# Mechanical Waves

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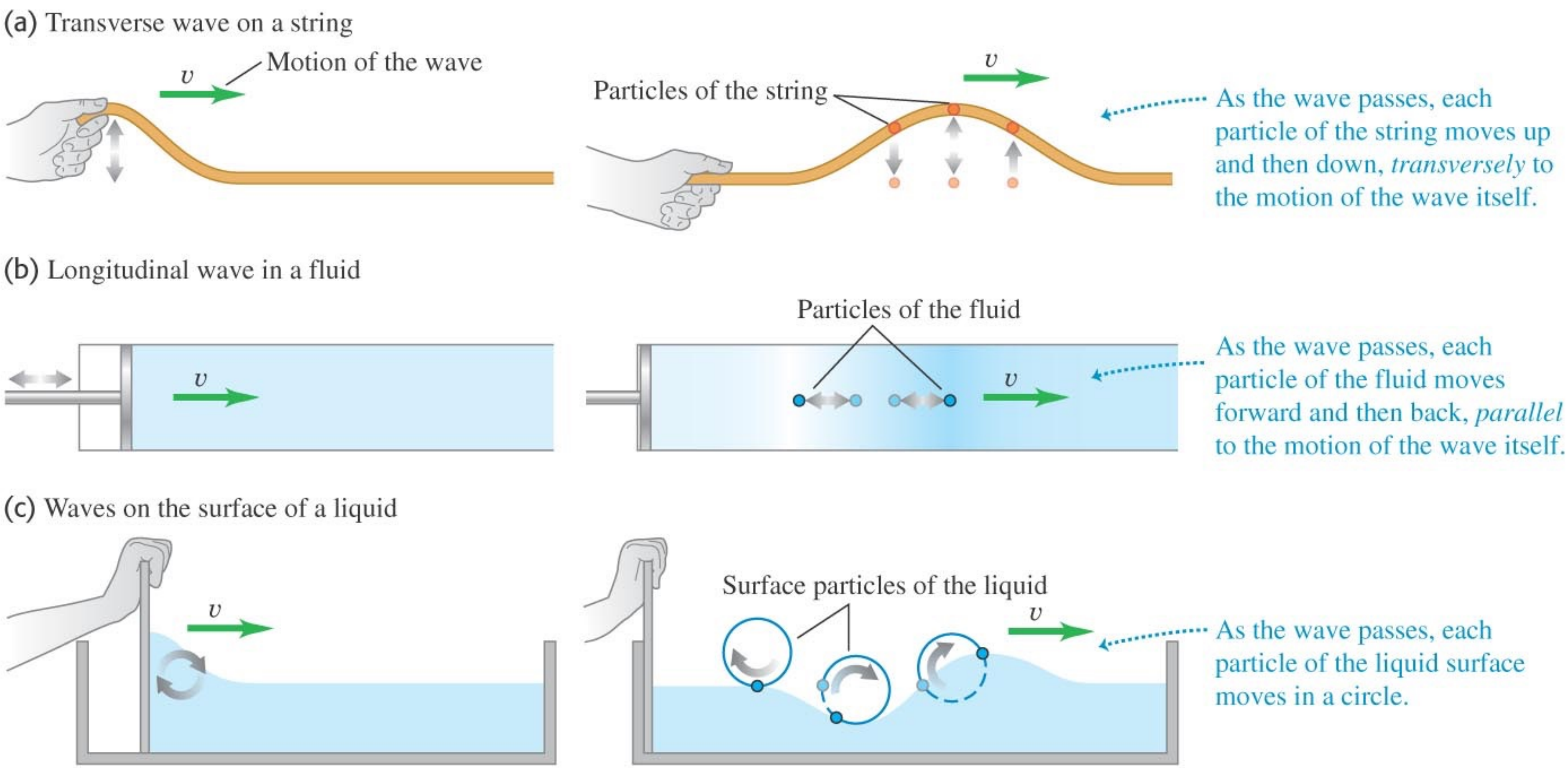
# Goals for Chapter 15

- To study the properties and varieties of mechanical waves
- To relate the speed, frequency, and wavelength of periodic waves
- To interpret periodic waves mathematically
- To calculate the speed of a wave on a string
- To calculate the energy of mechanical waves
- To understand the interference of mechanical waves
- To analyze standing waves on a string
- To investigate the sound produced by stringed instruments

# Types of mechanical waves

- A *mechanical wave* is a disturbance traveling through a *medium*.
- Figure below illustrates *transverse waves* and *longitudinal waves*.

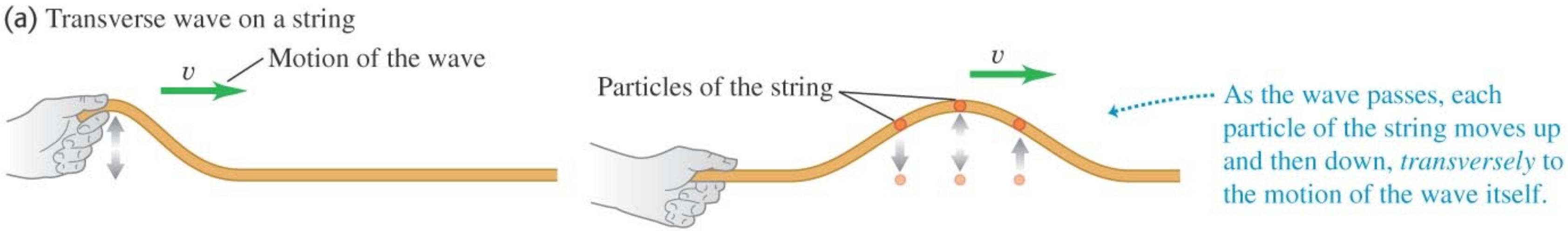


# PhEt simulations

http://phet.colorado.edu/sims/wave-on-a-string/wave-on-a-string_en.html

http://phet.colorado.edu/sims/normal-modes/normal-modes_en.html

# Some notes on waves




1. The waveform (disturbance) propagates with a definite speed through the medium. The speed is called the *wave speed*.

2. The medium does not travel through space. Individual particles undergoes periodic motions.

3. The wave transfers energy from one region to another.

Waves transport energy, but not matter, from one region to another.

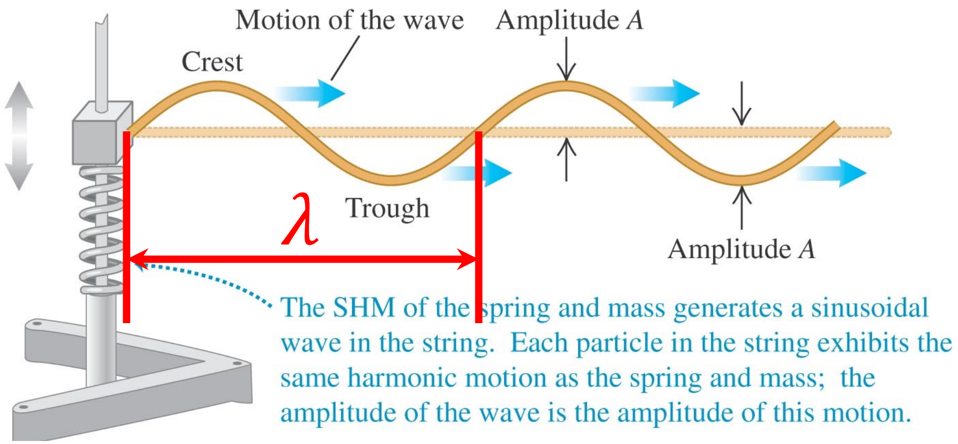
# Periodic waves

- For a *periodic wave*, each particle of the medium undergoes periodic motion.
- The *wavelength* $\lambda$ of a periodic wave is the length of one complete wave pattern.
- The speed of any periodic wave of frequency $f$ is

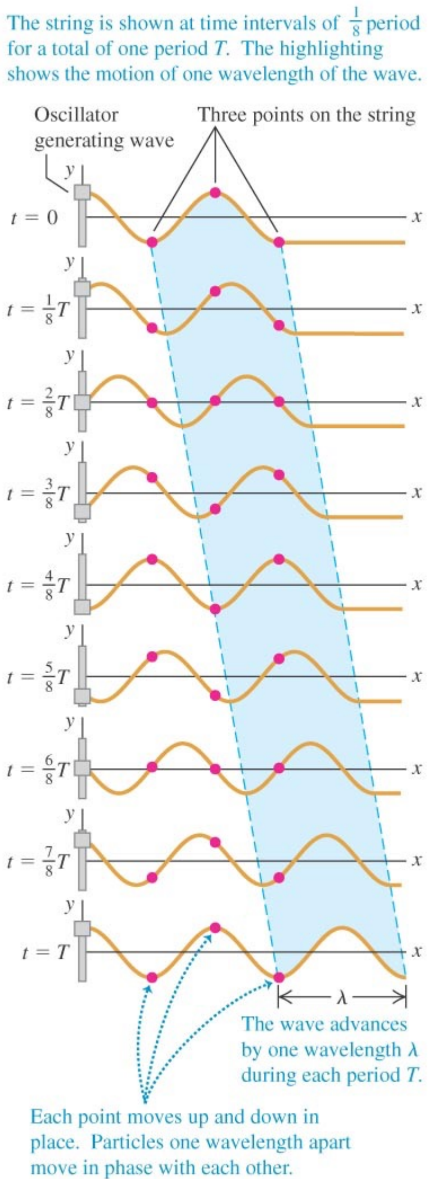
$$v = \lambda f.$$

# Periodic transverse waves

- For the transverse waves shown here, the particles move up and down, but the wave moves to the right.

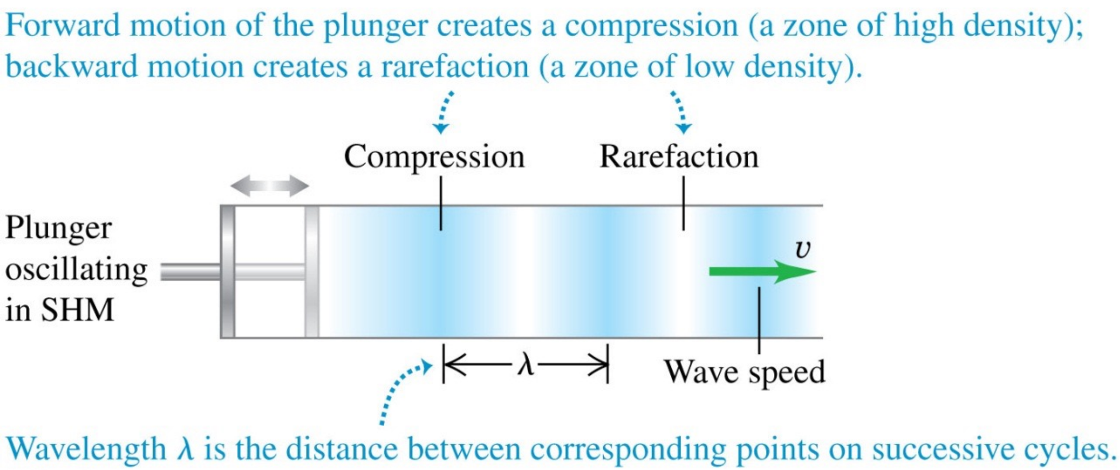


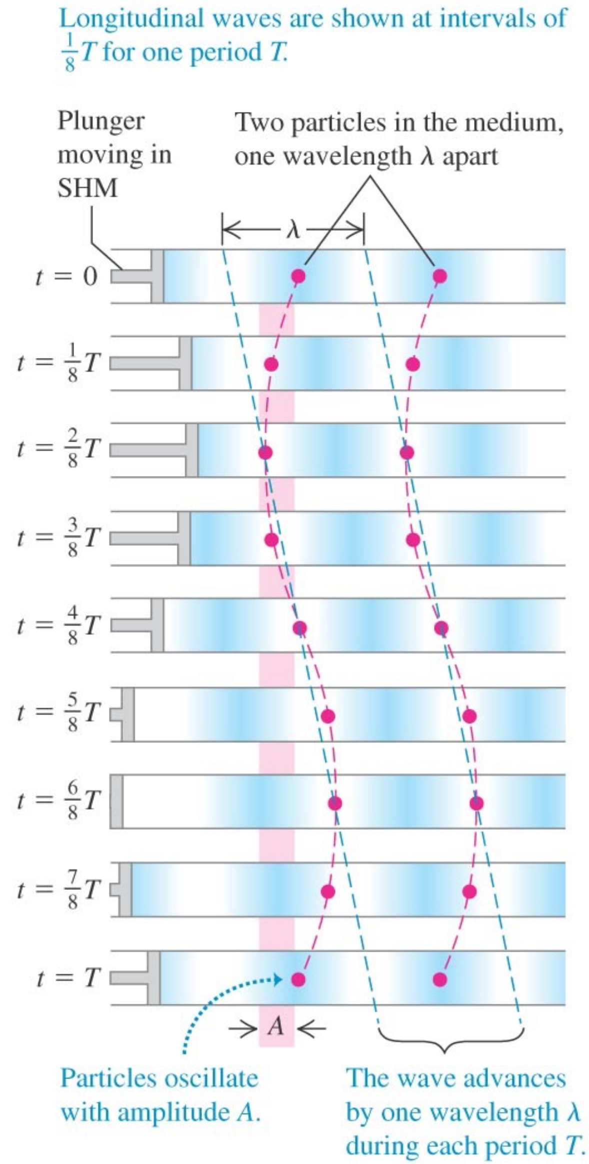

$$v = \frac{\lambda}{T} = \lambda f$$



# Periodic longitudinal waves

- For the longitudinal waves shown here, the particles oscillate back and forth along the same direction that the wave moves.



# Mathematical description of a Sinusoidal wave

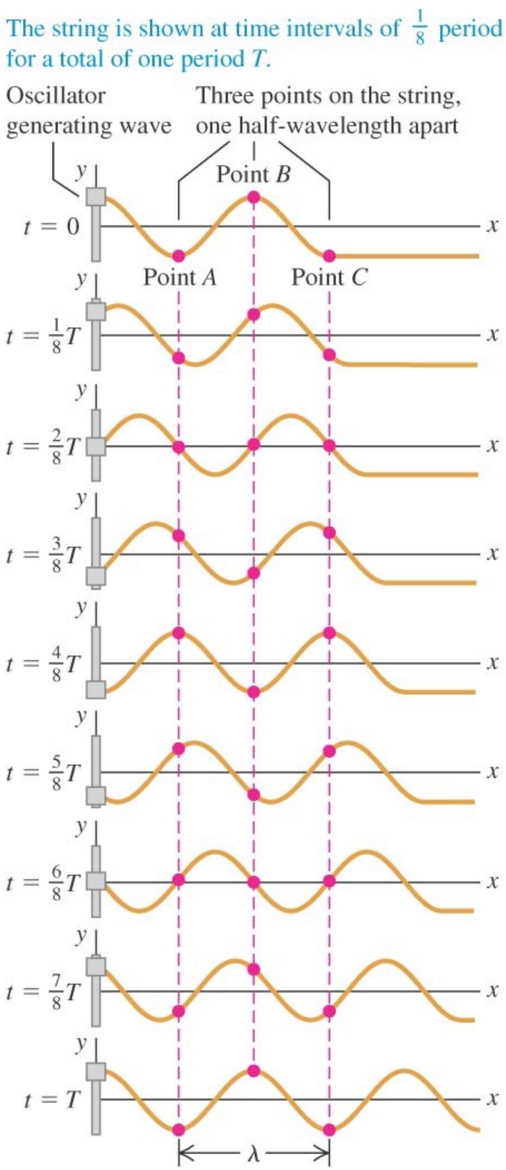
- The *wave function, y*(*x*,*t*), gives a mathematical description of a wave. In this function, *y* is the displacement of a particle at time *t* and position *x*.

- Considering the particle at $x = 0$, it undergoes SHM

$$y(x = 0, t) = A\cos\omega t$$

- The wave travels from $x = 0$ to $x$ in time $t = x/v$. The motion of point $x$ at time $t$ is the same of point $x = 0$ at time $t - x/v$

$$y(x,t) = A\cos\left[\omega\left(t - \frac{x}{v}\right)\right]$$



# Mathematical description of a Sinusoidal wave

Starting with

$$y(x,t) = A\cos\left[\omega\left(t - \frac{x}{v}\right)\right]$$

We have

$$y(x,t) = A\cos\left[\omega\left(\frac{x}{v} - t\right)\right] = A\cos[2\pi f\left(\frac{x}{v} - t\right)]$$

Using $T = 1/f$ and $\lambda = v/f$

$$y(x,t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

Define wave number $k = 2\pi/\lambda$ and use $\omega = \frac{2\pi}{T}$

$$y(x,t) = A\cos(kx - \omega t)$$

# Example:

1. A water wave traveling in a straight line on a lake is described by the equation $y(x,t) = (3.75\text{cm}) \cos(0.450\ \text{cm}^{-1}\ x + 5.40\ \text{s}^{-1}\ t)$, where y is the displacement perpendicular to the undisturbed surface of the lake.

(a) What's the amplitude?

(b) What's the wave number?

(c) What's the wavelength?

(d) What's the angular frequency?

(e) What's the frequency?

(f) What's the traveling speed of the wave?

$$y(x,t) = A\cos(kx - \omega t)$$

$$y(x,t) = (3.75\text{cm})\cos(-0.450\ \text{cm}^{-1}x - 5.40\ \text{s}^{-1}t)$$

(a) $A = 3.75\ \text{cm}$

(b) $k = -0.450\ \text{cm}^{-1}$

(c) $\lambda = \frac{2\pi}{|k|} = \frac{2\pi}{0.450\ \text{cm}^{-1}} = 14.0\ \text{cm}$

(d) $\omega = 5.40\ \text{Hz}$

(e) $f = \frac{\omega}{2\pi} = \frac{5.40\ \text{Hz}}{2\pi} = 0.860\ \text{Hz}$

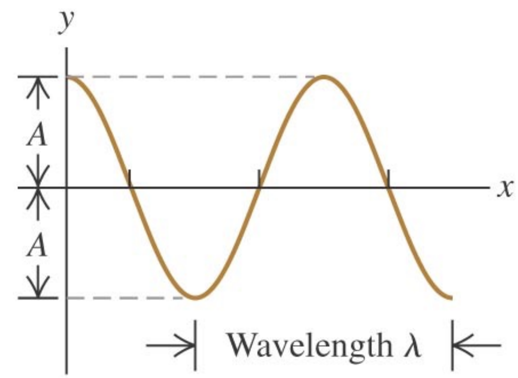
(f) $v = \lambda f = 14.0\ \text{cm} \times 0.860\ \text{Hz} = 12.0\ \text{cm/s}$

or $0.120\ \text{m/s}$.

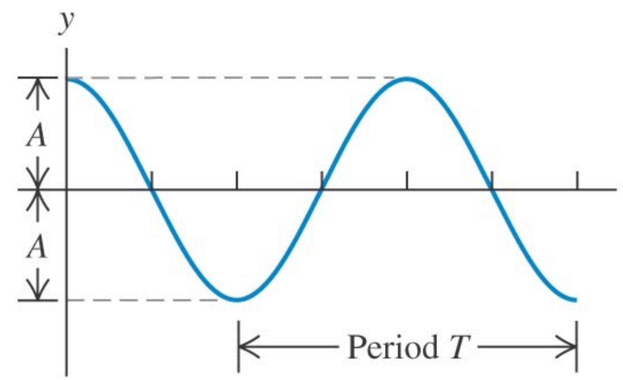
# Graphing the wave function

- The graphs in Figure to the right look similar, but they are *not* identical. Graph (a) shows the *shape* of the string at $t = 0$, but graph (b) shows the *displacement* $y$ as a function of time at $t = 0$.

(a) If we use Eq. (15.7) to plot $y$ as a function of $x$ for time $t = 0$, the curve shows the *shape* of the string at $t = 0$.




(b) If we use Eq. (15.7) to plot $y$ as a function of $t$ for position $x = 0$, the curve shows the *displacement* $y$ of the particle at $x = 0$ as a function of time.



# Particle velocity and acceleration in a sinusoidal wave

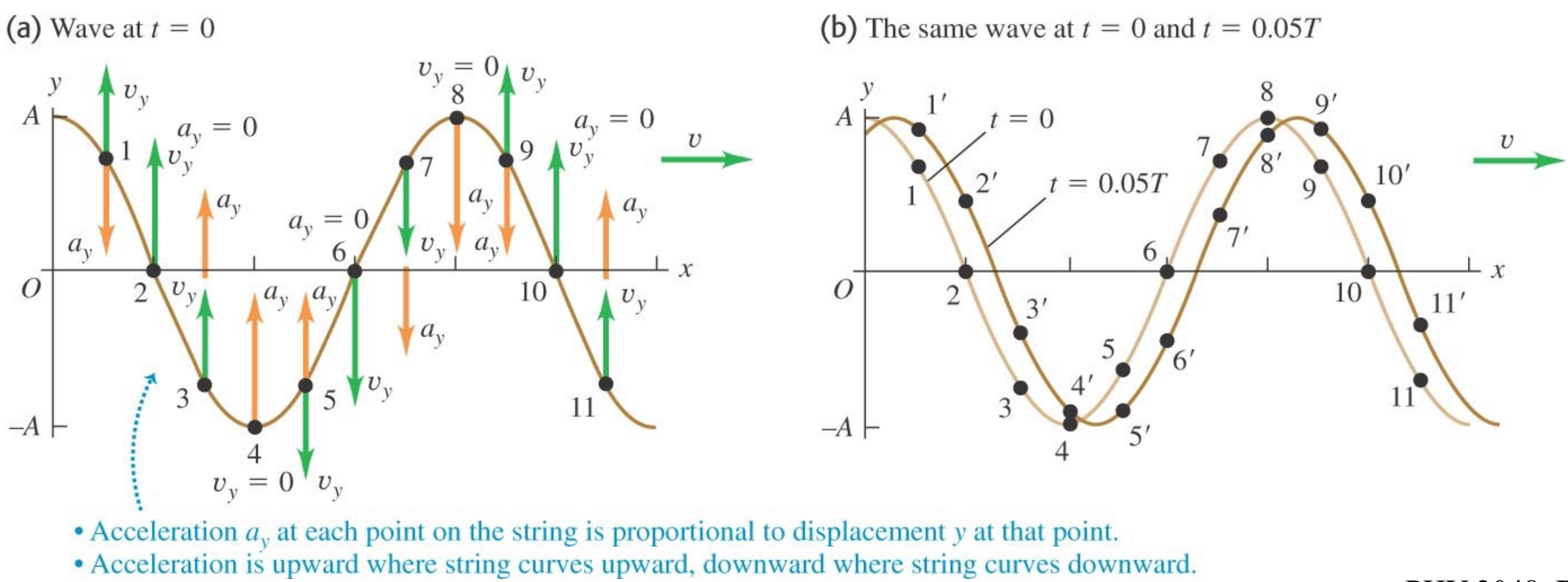
$$y(x,t) = A\cos(kx - \omega t)$$

Velocity:

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

Acceleration:

$$a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x,t)$$



# Wave equation

Using

$$a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x,t)$$

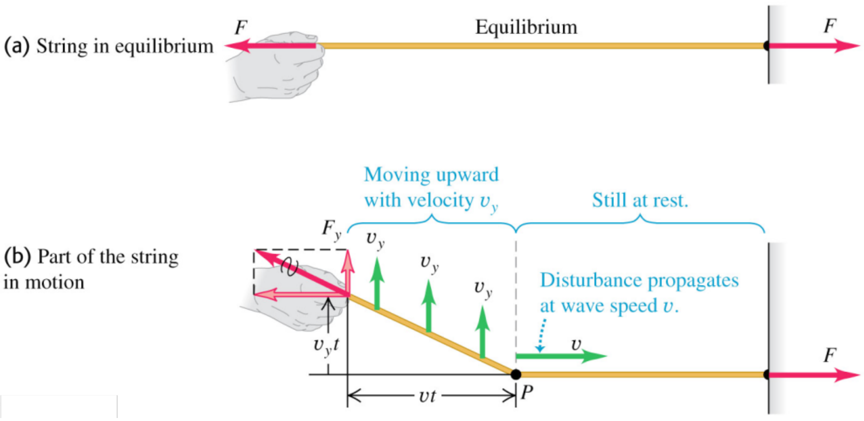
$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 A\cos(kx - \omega t) = -k^2 y(x,t)$$

$$\omega = vk$$

We obtain the *wave equation*

$$\boxed{\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}}$$

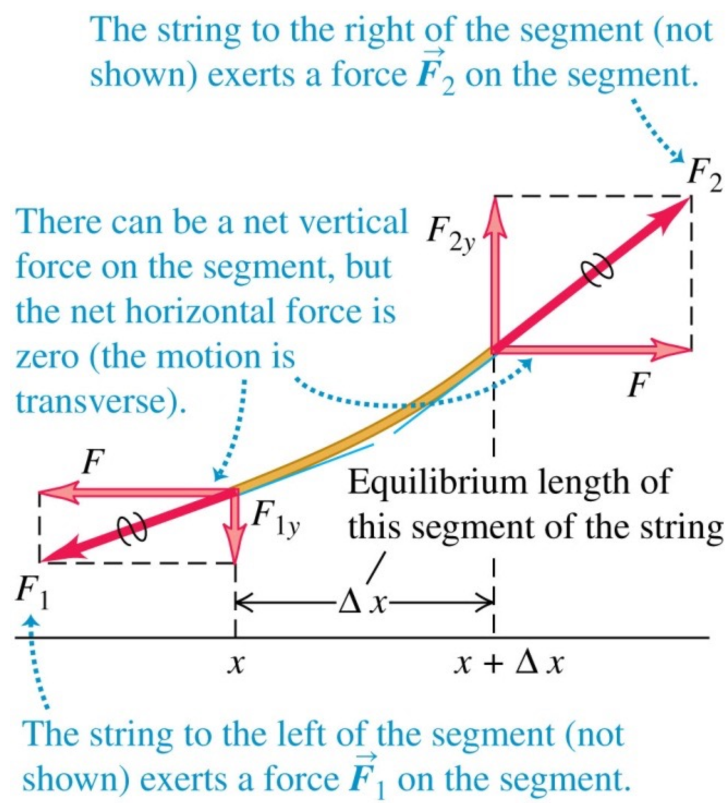
# The speed of a wave on a string




- Follow the first method on page 483 (using impulse-momentum theorem)

- Follow the second method on pages 484~485 (using Newton's second law)

$$v = \sqrt{\frac{F}{\mu}}$$



# Power in a wave

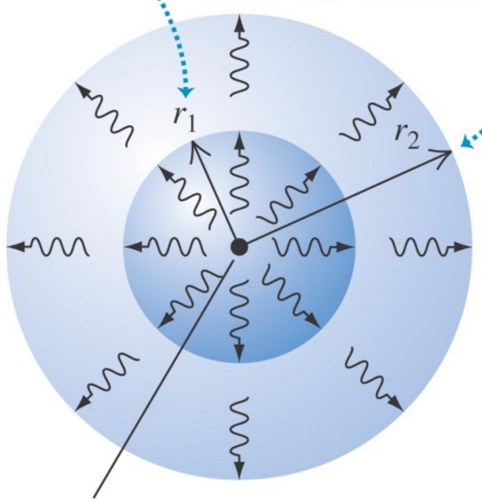
- A wave transfers power along a string because it transfers energy.
- The average power is proportional to the *square* of the amplitude and to the *square* of the frequency. This result is true for all waves.

$$P_{max} = \sqrt{\mu F}\,\omega^2 A^2$$

$$P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$

# Wave intensity

- The *intensity* of a wave is the average power it carries per unit area.
- If the waves spread out uniformly in all directions and no energy is absorbed, the intensity $I$ at any distance $r$ from a wave source is inversely proportional to $r^2$: $I \propto 1/r^2$.




$$I = \frac{P}{4\pi r^2}$$

$$P = 4\pi r_1^2 I_1 = 4\pi r_2^2 I_2 \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

# Example:

1. By measurement you determine that sound waves are spreading out equally in all directions from a point source and that the intensity is 0.050 W/m$^2$ at a distance of 4.0 m from the source.

(a) What is the power of the source?

(b) What is the intensity at a distance of 2.0 m from the source?

(a) $I = \frac{P}{4\pi r^2}$, $P = I 4\pi r^2 = 0.050\ W/m^2 \times 4\pi \times (4.0m)^2$

$= 3.2\pi W \approx 10.0\ W$

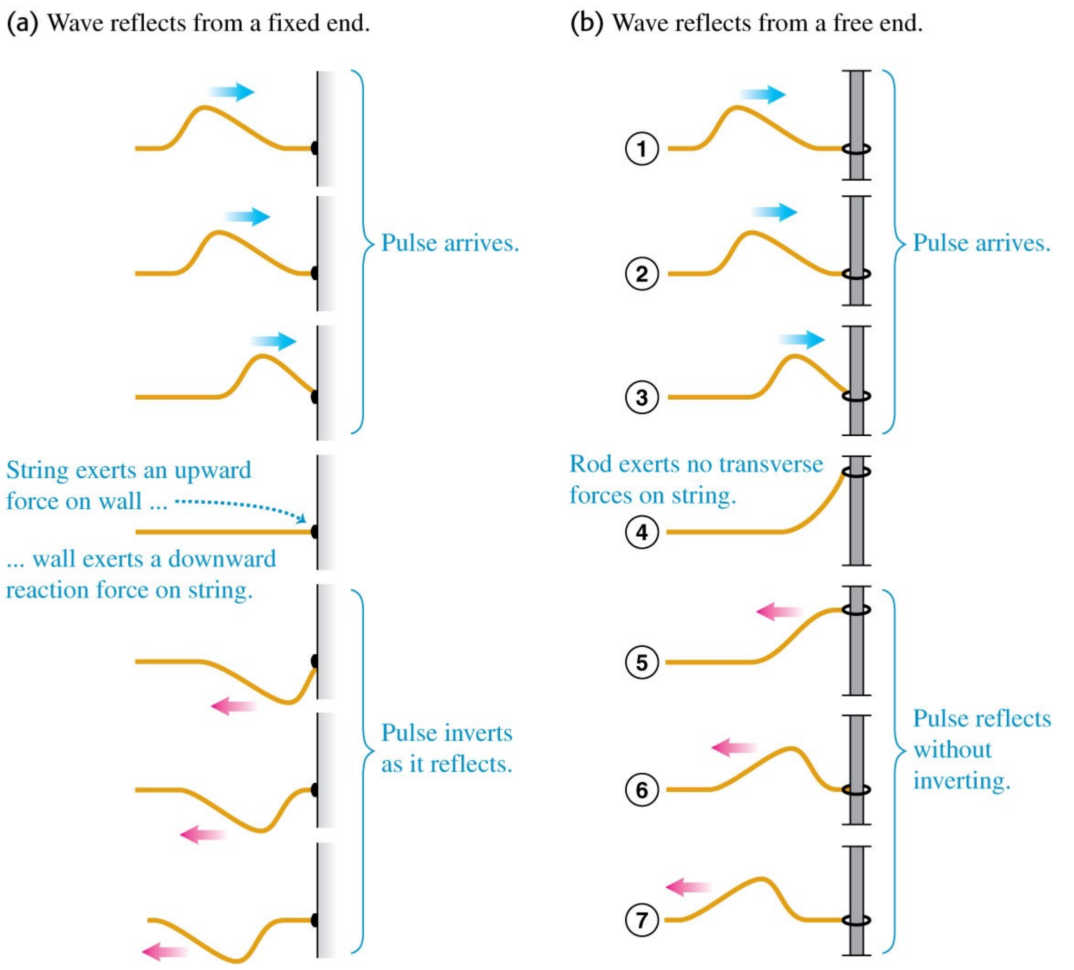
(b) $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$

$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 = \left(\frac{4.0m}{2.0m}\right)^2 \times 0.050\ W/m^2$

$= 0.200\ W/m^2$

# Boundary conditions

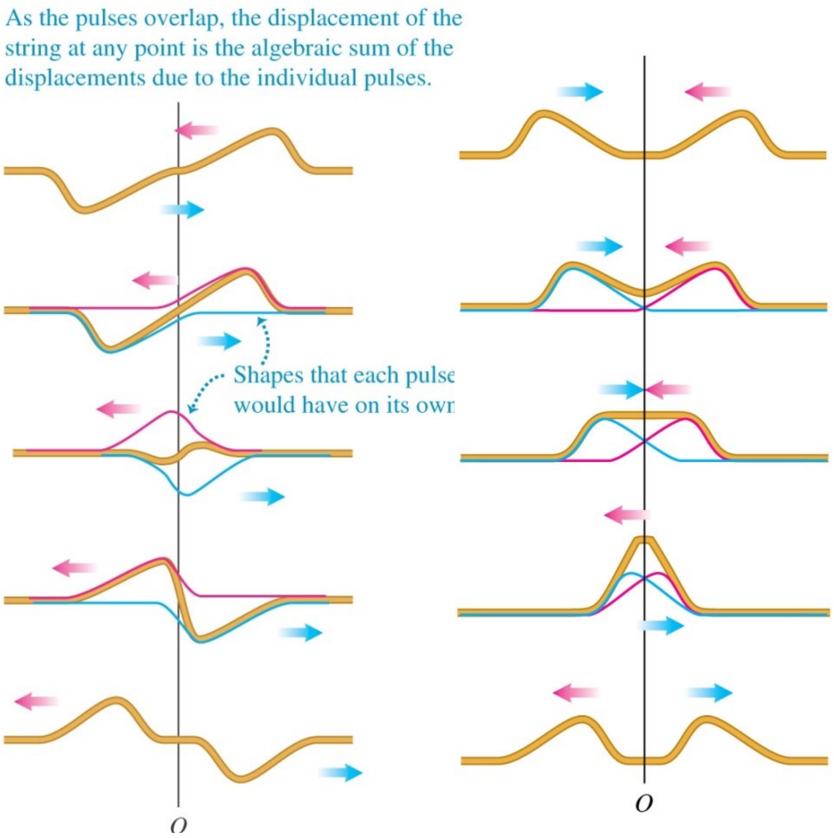
- When a wave reflects from a *fixed end*, the pulse *inverts* as it reflects. See Figure at the right.
- When a wave reflects from a *free end*, the pulse reflects *without inverting*. See Figure at the right.



# Wave interference and superposition

- *Interference* is the result of overlapping waves.
- *Principle of super-position*: When two or more waves overlap, the total displacement is the sum of the displace-ments of the individual waves.

$$y(x,t) = y_1(x,t) + y_2(x,t)$$



# The formation of a standing wave

- A wave to the left combines with a wave to the right to form a standing wave.

Incident wave traveling to the left

$$y_1 = -A\cos(kx + \omega t)$$

Reflected wave traveling to the right

$$y_2 = A\cos(-kx + \omega t)$$

The superposition of two waves

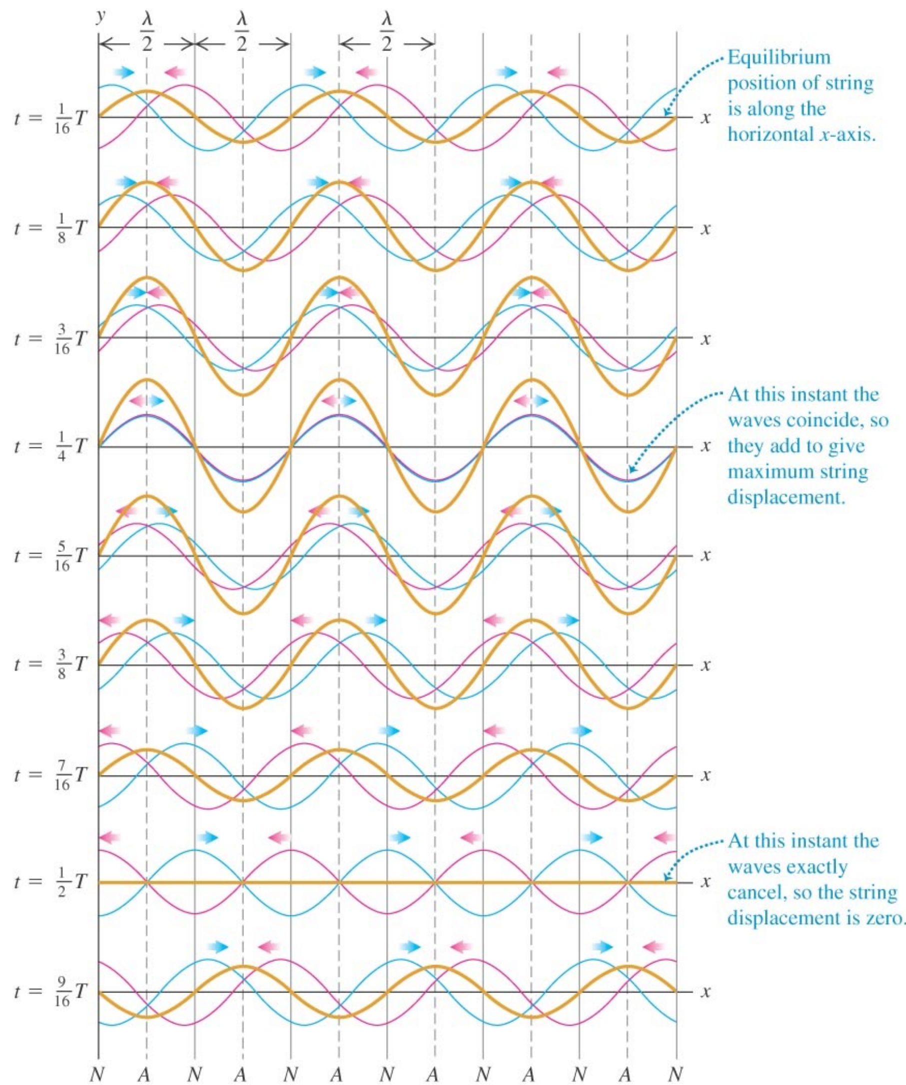
$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$= A[-\cos(kx + \omega t) + \cos(-kx + \omega t)]$$

$$= (2A\sin kx)\sin\omega t$$

When $\sin kx = 0$, or $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$, the amplitude is zero;

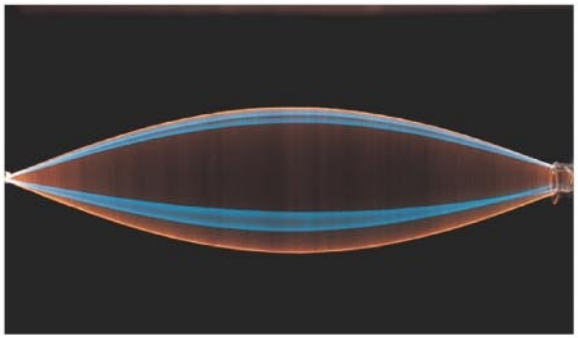
When $\sin kx = \pm 1$, or $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots$, the amplitude is maximum ($2A$);



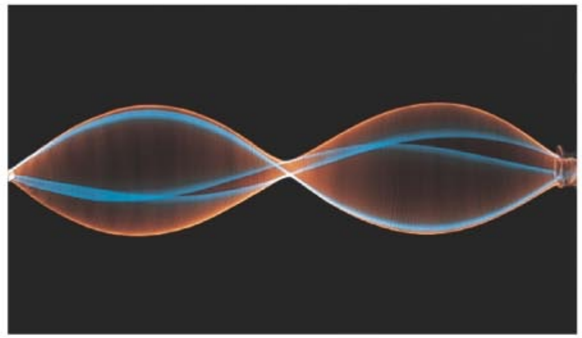
# Photos of standing waves on a string

- Some time exposures of standing waves on a stretched string.

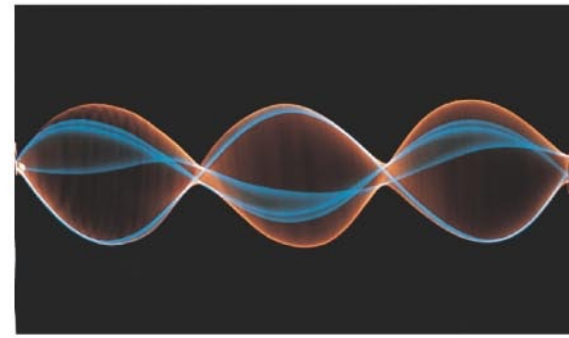
(a) String is one-half wavelength long.



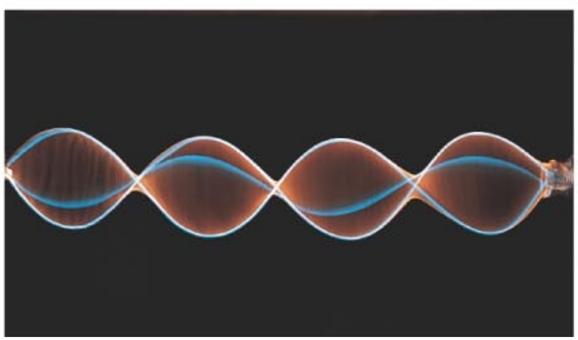
(b) String is one wavelength long.



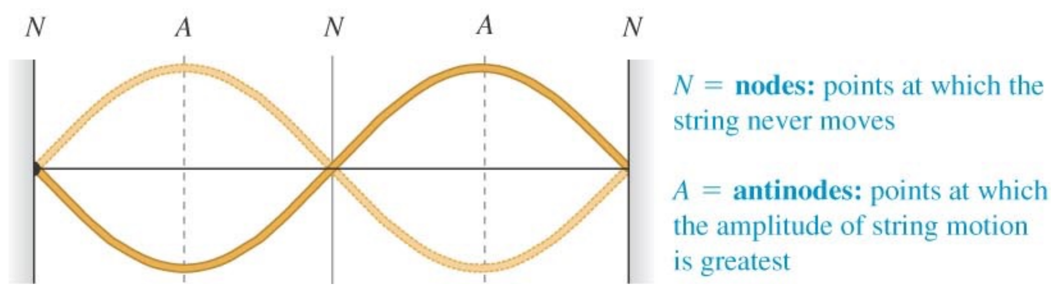
(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



(e) The shape of the string in (b) at two different instants



# Standing waves on a string

- Waves traveling in opposite directions on a taut string interfere with each other.
- The result is a *standing wave* pattern that does not move on the string.
- *Destructive interference* occurs where the wave displacements cancel, and *constructive interference* occurs where the displacements add.
- At the *nodes* no motion occurs, and at the *antinodes* the amplitude of the motion is greatest.

# Normal modes of a string

- For a taut string fixed at both ends, the possible wavelengths are

$$\lambda_n = 2L/n$$

and the possible frequencies are

$$f_n = n\, v/2L = nf_1,$$
$$\text{where } n = 1, 2, 3, \ldots$$

- $f_1$ is the *fundamental frequency*, $f_2$ is the second harmonic (first overtone), $f_3$ is the third harmonic (second overtone), etc.
- Figure illustrates the first four harmonics.