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Physics 2048 – Physics with Calculus 1

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USEFUL EQUATIONS AND NUMBERS:

Motion in One Dimension

Displacement $\Delta x = x_2 - x_1$

Average Velocity $v_{av} = \frac{\Delta x}{\Delta t}$ Instantaneous velocity $v(t) = \frac{dx}{dt}$

Average acceleration $v_{av} = \frac{\Delta v}{\Delta t}$ Instantaneous acceleration $a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Velocity at given time $v = v_0 + at$ Position at given time $x = x_0 + v_0 t + \frac{1}{2}at^2$

Motion in Two and Three Dimensions

Vector components $A_x = Acos(\theta)$, $A_y = Asin(\theta)$

Vector magnitude $A = \sqrt{A_x^2 + A_y^2}$

Position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Instantaneous velocity $\vec{v}(t) = \frac{d\vec{r}}{dt}$ Instantaneous acceleration $\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

Velocity at given time $\vec{v} = \vec{v}_0 + \vec{a}t$ Position at given time $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

Equations through the x components: $v_x = v_{0x} + a_x t$, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

Equations through the y components: $v_y = v_{0y} + a_y t$, $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

Newton's Law

Second Law $\sum \vec{F} = m\vec{a}$

Through x projections: $\sum F_x = ma_x$ Through y projections: $\sum F_y = ma_y$

Weight $\vec{w} = m\vec{g}$; $g = 9.81 m/s^2$ Hook's law $F_x = -k\Delta x$

Applications of Newton's Law

Maximal Static friction: $f_{s,max} = \mu_s F_n$ Static Frication $f_s \leq \mu_s F_n$

Kinetic Friction: $f_k = \mu_k F_n$

Work and Energy

Work: constant force $W = Fcos\theta\Delta x$ variable force $\int\limits_{x_1}^{x_2} F_x dx$

Work: constant Force in three dimensions $W = \vec{F}\cdot\vec{s}$ variable force $\int\limits_{1}^{2} \vec{F}\cdot\vec{ds}$

Kinetic Energy $K = \frac{1}{2}mv^2$

Work-Kinetic Energy Theorem $W_{total} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Dot Product $\vec{A}\cdot\vec{B} = ABcos\theta$

Power $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$ Potential Energy $dU = -\vec{F}\cdot\vec{ds}$

Gravitational potential energy $U = mgy$ Potential energy of spring $U = \frac{1}{2}Kx^2$

Conservation of Energy

Mechanical Energy $E_{mech} = K + U$

Conservation of mechanical energy $K + U = const$

$K_f + U_f = K_i + U_i$

Systems of Particles and Conservation of Momentum

Center of mass: $M\vec{r}_{cm} = \sum\limits_i m_i\vec{r}_i$

Motion of center of mass: $\vec{F}_{net,ext} = M\vec{a}_{cm}$

Momentum $\vec{p} = m\vec{v}$ $K = \frac{p^2}{2m}$

When $F_{net,ext} = 0$ $\sum\vec{p}_i = const$

Rotation

Angular velocity: $\omega = \frac{d\theta}{dt}$ Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Tangential speed: $v = r\omega$ Tangential acceleration: $a = r\alpha$

Centripetal acceleration: $a_c = \frac{v^2}{r} = r\omega^2$

Torque: $\tau = Fl$ Moment of Inertia: $I = \sum m_i r_i^2$

Moment of inertia of uniform disk: $I = \frac{1}{2}MR^2$, hoop $I = MR^2$, solid sphere $I = \frac{2}{5}MR^2$

Newton's Second Law for rotation: $\tau_{net,ext} = I\alpha$

Kinetic energy: $K = \frac{1}{2}I\omega^2$

Conservation of Angular Momentum

Vector Nature of Rotation: velocity $\vec{\omega}$ Torque $\vec{\tau} = \vec{r}\times\vec{F}$

Vector product: $\vec{A}\times\vec{B} = ABsin\phi\hat{n}$

Angular momentum $\vec{L} = \vec{r}\times\vec{p}$

Conservation of angular momentum: if $\tau_{net,ext} = 0$ $L_{sys} = const$

Gravity

Newton's Law of Gravity: magnitude $F = \frac{Gm_1m_2}{r_{12}^2}$

vector force $\vec{F} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}$

$G = 6.67 \times 10^{-11} Nm^2/kg^2$

Gravitational potential energy: $U(r) = -\frac{GMm}{r}$, $U = 0$ at $r \to \infty$

$g = \frac{GM_E}{R_E^2}$; radius of the earth: $R_E = 6.37 \times 10^6$m

Escape Velocity: $v_e = \sqrt{\frac{2GM_E}{(R_E+h)}}$

Oscillations

Position function: $x = Acos(\omega t + \delta)$ $\quad \omega = 2\pi f = 2\pi/T$

Total energy: $E_{total} = \frac{1}{2}kA^2$ $\quad K_{AV} = U_{AV} = \frac{1}{2}E_{total}$

Period: for spring $T = 2\pi\sqrt{\frac{m}{k}}$ $\quad$ for simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

Wave Motion

Wave function: $y(x,t) = Acos(kx \pm \omega t + \delta)$

$\omega = 2\pi f = 2\pi/T$ $\quad k = \frac{2\pi}{\lambda} \quad v = f\lambda$

Speed of waves on a string $v = \sqrt{\frac{F}{\mu}}$ $\quad$ sound waves $v = \sqrt{\frac{B}{\rho}}$, speed of sound: 340 m/s

Some Trigonometric Relations

$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\cos(90-\alpha) = \sin(\alpha); \quad \cos(180-\alpha) = -\cos(\alpha)$

$\sin(90-\alpha) = \cos(\alpha); \quad \sin(180-\alpha) = \sin(\alpha)$

$1-\cos(\alpha) = 2\sin^2(\frac{\alpha}{2}); \quad \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$