Lecture 12

Tuesday, May 30, 2023 10:07 AM

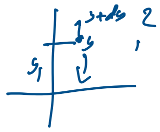
Principle or Physical Laws?

Leonhard Euler 1707 – 1783

Joseph-Lois Lagrange 1736 – 1813

– Minimal Action Principle

Gravity $U = mgh$



$U_1 = mgy$

$U_2 = mg(y+dy)$

$\vec{W} = \vec{F}_g = -mg$

$dU = U_2 - U_1 = mg(y+dy) - mgy = mg\,dy$

$$\boxed{\frac{dU}{dy} = mg = -\vec{F}_g}$$

Spring $U = \frac{1}{2}kx^2$

$\vec{F}_s = -kx$

$$\frac{dU}{dx} = \frac{d(\frac{1}{2}kx^2)}{dx} = kx = -\vec{F}_s$$

$$\frac{\vec{d}U}{d\vec{r}} = -\vec{F}$$

$$\boxed{\vec{\nabla} U = -\vec{F}}$$

$\text{grad}\,\vec{U} = -\vec{F}$

gradient

What is Reality

– Nature Exist without us

Reality

– Principle

1. Function Lagrange-Euler Function / Lagrangian

$K = \frac{1}{2}mv^2$ $L(\vec{r}, t) = K - U$

2. Action $S = \int_{t_1}^{t_2} L(\vec{r}, t)\,dt$

3. $\delta S = 0$ – minimal (Principle)

All Laws of Mechanics

⟹ Electromagnetism — laws
→ Laws

① Lagrangian $L(q, t)$
18'th ② Action
③ Minimiz Action

20'th Nucler Force $L(x, t)$
1
2
3

Weak Force $L(x, t)$
1
2
3

Antropic Principle !

Chapter 9: Rotation of Rigid Bodies

⟹ Physics of Motion

1) Linear Motion
One Point object

1d-motion
2d-motion

Kinematics (Equation of Motion) Galileo
Dynamics (Newton's 3 Laws)

Work, Energy, — Kinetic Energy
— Potential Energy

Conservation of Total Mechanical Energy

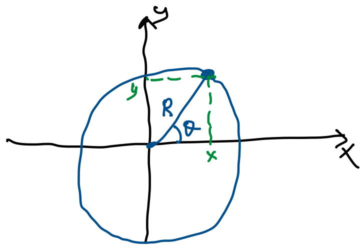
2) Linear Motion of systems of Point objects
(Extended Objects)

Center of Mass, Momentum of P.O.
Momentum Conservation $\sum_{i=1}^{N} P_i = \text{const}$

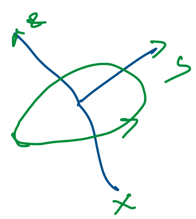
3) Rotational Motion

- One Point Object
- System of Point Object
- Extended Objects } Rotation of Rigid Bodies

⇒ Rotation

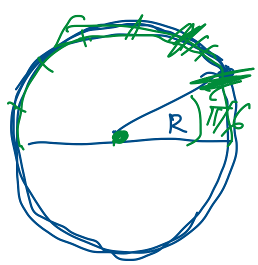


1. Rotation of P.O.
2. Axis of Rotation } → x
3. Radius
4. θ — + CounterClockwise; − Clockwise



Neutrino

4' θ — degrees:



$\theta_{Full} = 360°$ | $180°$

$\ell = 2\pi R$ | $\ell' = \pi R$

$\frac{\ell}{R} = 2\pi$ | $\frac{\ell'}{R} = \pi$

Radians

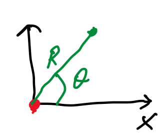
30 degr

$1\,deg° = \frac{2\pi}{360}$ radians

$\boxed{30\,deg = 30 \times 1\,deg = 30 \cdot \frac{2\pi}{360} \text{ radian}}$

- 1d Motion
1) Linear Motion of P.O.
2) Reference Frame with reference Point (− 0 + x)
3) Position of P.O. x, m ±
3') Time t, s +
4) Displacement $\Delta x = x_f - x_i$
4') Time Interval $\Delta t = t_f - t_i$
5) Average Velocity $V_{av} = \frac{\Delta x}{\Delta t}$, $\frac{m}{s}$, ±

- Rotation
1) Rotation of P.O
2) Rotation Axis with Reference Direction



3) Position of P.O. R, θ } θ, rad, ±
3') Time t, s +
4) Angular Displacement $\Delta\theta = \theta_f - \theta_i$, rad, ±
4') Time Interval $\Delta t = t_f - t_i$
5) Average Angular Velocity $\omega_{av} = \frac{\Delta\theta}{\Delta t}$, $\frac{rad}{s}$, ±

$$V = \frac{dx}{dt}, \ \frac{m}{s}, \ \pm \qquad\qquad \omega = \frac{d\theta}{dt}$$

6) Average Acceleration $a_{AV} = \frac{\Delta V}{\Delta t}, \ \frac{m}{s^2}, \pm$ 6) Average Angular Acceleration $\alpha_{AV} = \frac{\Delta \omega}{\Delta t}, \ \frac{rad}{s^2} \pm$

$$a = \frac{dV}{dt} \qquad\qquad \alpha = \frac{d\omega}{dt},$$

Equation of Motion

a) Baby Version $a = 0$
(running fine)

$$V(t) = V_i = const$$
$$X(t) = X_i + V \cdot t$$

b) Junior Version $a = const$

$$V(t) = V_i + at$$
$$X(t) = X_i + V_i t + \frac{at^2}{2}$$
$$V_f^2 - V_i^2 = 2a\Delta X$$

c) Adult Version $a \neq const$

$$V(t) = \int_i^f a(t)\,dt$$
$$X(t) = \int_i^f V(t)\,dt$$

Equation of Motion

a) Baby Version $\alpha = 0$

$$\omega(t) = \omega_i = \omega = const$$
$$\theta(t) = \theta_i + \omega \cdot t$$

b) Junior Version $\alpha = const$

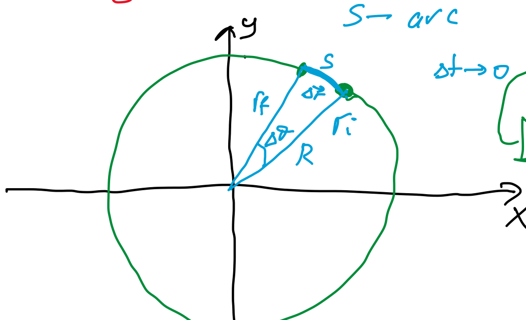
$$\omega(t) = \omega_i + \alpha t$$
$$\theta(t) = \theta_i + \omega_i t + \frac{\alpha t^2}{2}$$
$$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$$

c) Adult Version

$$\omega(t) = \int_i^f \alpha(t)\,dt$$
$$\theta(t) = \int_i^f \omega(t)\,dt$$

⇒ Relating Linear and Rotational (Angular) Motions

S – arc $\qquad \Delta\vec{r} = \vec{r}_f - \vec{r}_i$



$\Delta t \to 0$

$$\boxed{S \approx \Delta r}$$
$$\boxed{S = R \cdot \Delta\theta}$$
$$\boxed{\frac{\Delta r}{\Delta t} = R\frac{\Delta\theta}{\Delta t}}$$
$$\boxed{V = R\omega}$$

$$\boxed{\Delta r = R \cdot \Delta\theta \quad \boxed{V = R\omega}}$$

$$\frac{dV}{dt} = R\frac{d\omega}{dt}$$

$$\boxed{a = R\alpha} \qquad \boxed{a = K\alpha}$$

⇒ Energy of Rotational Motion

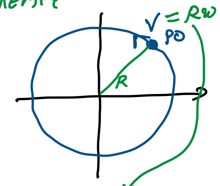
⇒ Kinetic Energy of rotating Point object?

Rotational Mass   Moment of Inertia

$$\boxed{K^{Rot} = \frac{1}{2} I \omega^2} \qquad \boxed{I = mR^2}$$

Kin Energy of Linear Motion

$$\boxed{K = \frac{1}{2} m v^2}$$



$$K = \frac{1}{2} m v^2$$

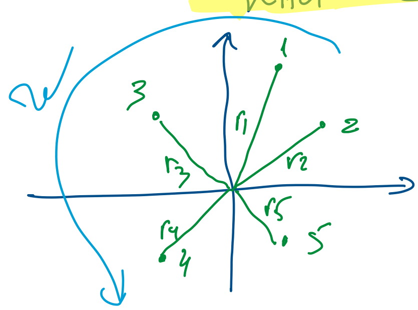
$$\boxed{K = \frac{1}{2} m (R\omega)^2 = \frac{1}{2} mR^2 \cdot \omega^2}$$

⇒ Summerizing

$$K = \frac{1}{2} I \omega^2, \quad I = mR^2$$

– Rotational Kinetic Inergy of Point Object

⇒ Kinetic Energy of system of P.O. Rotating with same ω



$$K_1 = \frac{1}{2} m_1 r_1^2 \omega^2$$
$$K_2 = \frac{1}{2} m_2 r_2^2 \omega^2$$
$$K_3 = \frac{1}{2} m_3 r_3^2 \omega^2$$
$$K_4 = \frac{1}{2} m_4 r_4^2 \omega^2$$
$$K_5 = \frac{1}{2} m_5 r_5^2 \omega^2$$

$$K_{Tot} = K_1 + K_2 + K_3 + K_4 + K_5 = \frac{1}{2}\left[m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 + m_5 r_5^2\right]\omega^2$$

$$I^{sys} = \sum_{i=1}^{5} m_i r_i^2$$

⇒

$$\boxed{\begin{gathered} K = \frac{1}{2} I^{sys} \omega^2 \\ I^{sys} = \sum_{i=1}^{N} m_i r_i^2 \end{gathered}}$$