Lecture 13

Wednesday, May 31, 2023 10:00 AM

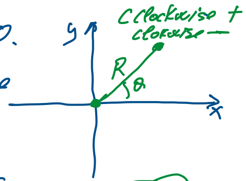
Chapter 9: Rotation of Rigid Bodies

Kinematics of Rotation

1. Rotation of P.O.
2. Reference Frame

Reference direction

Axis of Rotation, θ



3. Position in the Space

R, θ — radians; R: m; θ: +, −

$\frac{2\pi R}{R} = 2\pi$ radians

$1\,deg = \frac{2\pi}{360}$ rad.

3' Position in time t, s, +

4 Angular Displacement $\Delta\theta = \theta_f - \theta_i$, rad, $\pm$

4' Time Interval $\Delta t = t_f - t_i$, s $\pm$

5 Average Angular Velocity $\omega_{av} = \frac{\Delta\theta}{\Delta t}$, $\frac{rad}{s}$, $\pm$

Instantaneous angular velocity — $\omega = \frac{d\theta}{dt}$, $\frac{rad}{s}$

6. Average Acceleration $\alpha = \frac{\Delta\omega}{\Delta t}$, $\frac{rad}{s^2}$, $\pm$

Instantaneous Acceleration $\alpha = \frac{d\omega}{dt}$

Equation of Motion

(a) Baby Version $\alpha = 0$

$$\omega(t) = \omega_i = \omega = const$$

$$\theta(t) = \theta_i + \omega t$$

(b) Junior version $\alpha = const$

$$\omega(t) = \omega_i + \alpha t$$

$$\theta(t) = \theta_i + \omega_i t + \frac{\alpha t^2}{2}$$

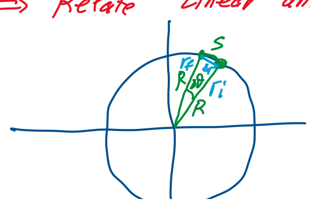
$$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$$

(c) Adult Version $\alpha \neq const$

$$\omega(t) = \int_i^f \alpha(t)\,dt$$

$$\theta(t) = \int_i^f \omega(t)\,dt$$

⟹ Relate Linear and Rotational Motion



$s = R\Delta\theta$

$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$\Delta t \to 0$

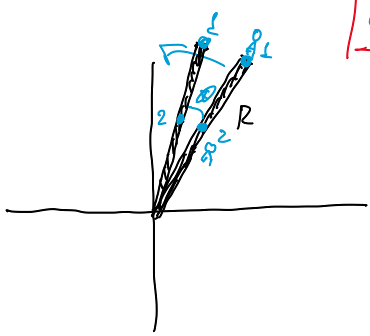
$\boxed{s \approx \Delta r}$

$$\frac{\Delta r}{\Delta t} = v = R\frac{\Delta\theta}{\Delta t} = R\omega$$

$$\boxed{V = R\omega}$$

$$\frac{dV}{dt} = R\frac{d\omega}{dt}$$

$$\boxed{a = R\alpha}$$



ω_1 $\qquad$ ω_2

$\omega_1 = \frac{\Delta\theta_1}{\Delta t}$ $\qquad$ $\omega_2 = \frac{\Delta\theta_2}{\Delta t}$

$\Delta\theta_1 = \Delta\theta_2 = \Delta\theta$

$\omega_1 = \frac{\Delta\theta}{\Delta t} = \omega_2 = \frac{\Delta\theta}{\Delta t} = \omega$

$V = R \cdot \omega$

$V_1 = R \cdot \omega$ $\qquad$ ① $a_c^1 = \frac{V^2}{R} = \frac{R^2\omega^2}{R} = R\omega^2$

$V_2 = \frac{R}{2}\omega$ $\qquad$ ② $a_c^2 = \frac{V_2^2}{R} = \frac{(R/2)^2\omega^2}{R} = \frac{R\omega^2}{4}$

$\alpha_1 = 0 \quad \alpha_2 = 0 \qquad \underline{\omega = \text{const}}$

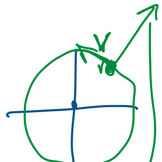
⟹ Energy of Rotational Motion

— Point Object

Linear Motion $\quad K = \frac{1}{2}mV^2$

Rotational Mass

Rotation



$K = \frac{1}{2} I \omega^2$

$V = R\omega$

$K = \frac{1}{2}mV^2 = \frac{1}{2}mR^2\omega^2$

$$\boxed{K = \frac{1}{2}I\omega^2}$$

Rotational Mass, P.O.

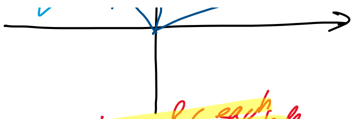
$$\boxed{I = mR^2}$$ — Moment of Inertia

⟹ Kinetic Energy of system of P. O.

Rotating with same ω

$V_1 = r_1\omega \qquad V_2 = r_2\omega \qquad V_3 = r_3\omega$

$K^{Tot} = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 + \frac{1}{2}m_3V_3^2$



$$K^{TOT} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2$$

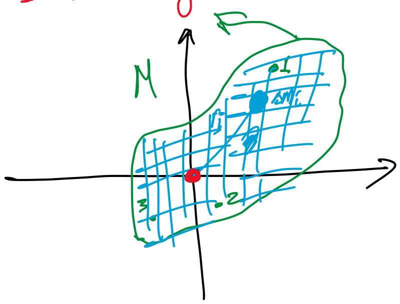
$$= \frac{1}{2} \underbrace{\left(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2\right)}_{I^{sys}} \omega^2$$

ω = same for each particle

$$\boxed{I^{sys} = \sum_{i=1}^{N} m_i r_i^2}$$

$$\boxed{K = \frac{1}{2} I^{sys} \omega^2}$$

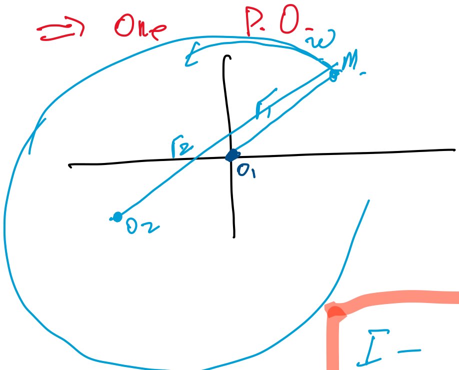
⇒ Rigid Bodies ω_1 ω_2 ω_3



$K =$

$$\boxed{I} = \sum_{i}^{N} r_i^2 \Delta m_i \Big|_{\Delta m \to 0} = \boxed{\int_{volume} r^2 dm}$$

⇒ One P.O.



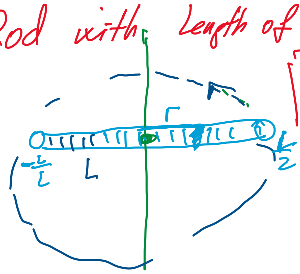
$K = \frac{1}{2} I \omega^2$

$I_1 = m r_1^2$

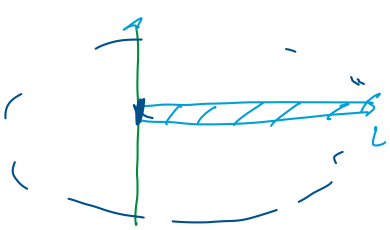
$I_2 = m r_2^2$

I — depends on where is an axis of Rotation

- Rod with Length of L

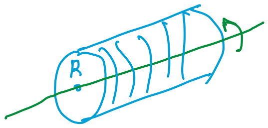


$$\boxed{I} = \int_{-\frac{L}{2}}^{\frac{L}{2}} r^2 dm = \boxed{\frac{1}{12} M L^2}$$



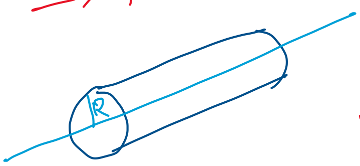
$$\boxed{I = \frac{1}{3} M L^2}$$

⇒ Solid cylinder



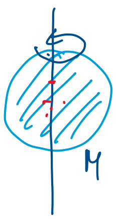
$$I = \frac{1}{2} M R^2$$

⇒ Hollow Cylinder



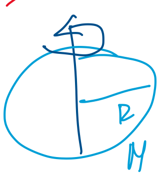
$$I = M R^2$$

⇒ Solid Sphere



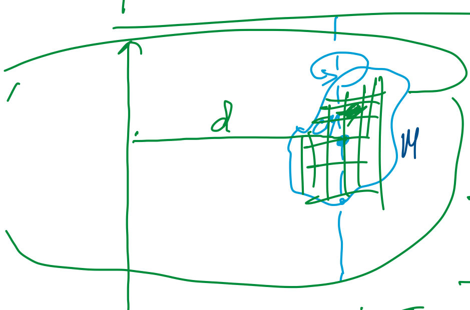
$$I = \frac{2}{5} M R^2$$

⇒ Hollow Sphere



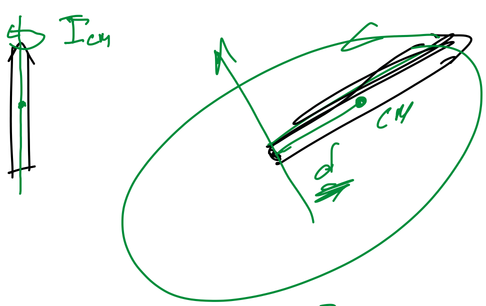
$$I = \frac{2}{3} M R^2$$

⇒ Parallel - Axis Theorem



I_{cm} = given

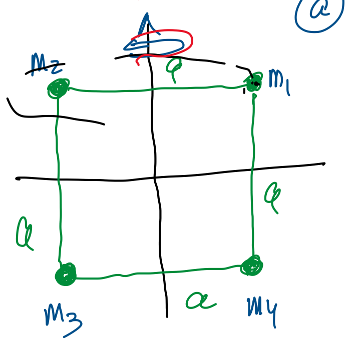
$$I = I_{cm} + M d^2$$



$$I = I_{cm} + M d^2$$

Examples:

(a)



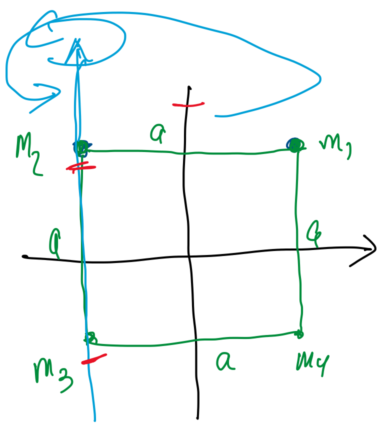
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$r_1 = \frac{a}{2} \qquad r_2 = \frac{a}{2}$$

$$r_3 = \frac{a}{2} \qquad r_4 = \frac{a}{2}$$

$$\boxed{I} = \overbrace{(m_1 + m_2 + m_3 + m_4)}^{4m}\left(\frac{a}{2}\right)^2 = 4m\frac{a^2}{4} = \boxed{ma^2}$$

$m_1 = m_2 = m_3 = m_4$



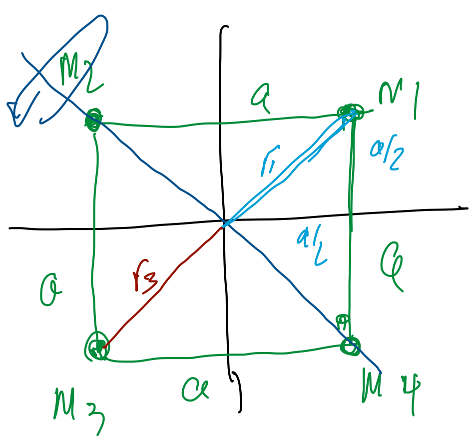
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$r_1 = a \qquad r_2 = 0 \qquad r_3 = 0$$

$$r_4 = a$$

$$\boxed{I} = m_1 a^2 + m_4 a^2 = (m_1 + m_2)a^2 = \boxed{2ma^2}$$

$m_1 = m_2 = m$



$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$r_1 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$$

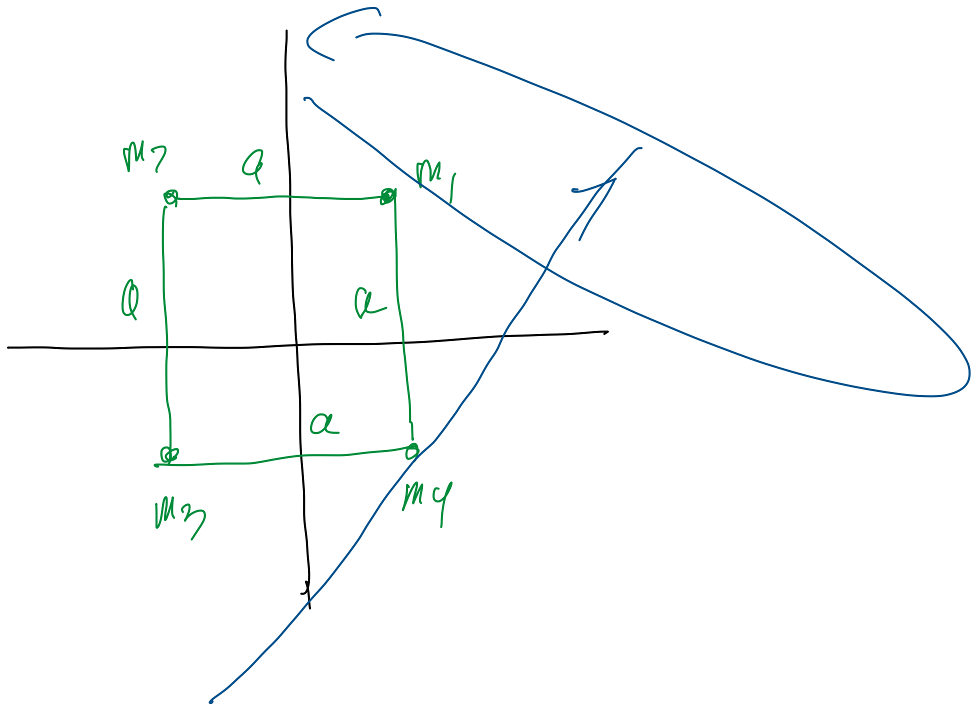
$$r_2 = 0$$

$$r_3 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$$

$$r_4 = 0$$

$$\boxed{I} = m_1 \frac{a^2}{2} + m_3 \frac{a^2}{2} = (m_1 + m_2)\frac{a^2}{2} =$$

$$m_1 = m_2 \Rightarrow m$$



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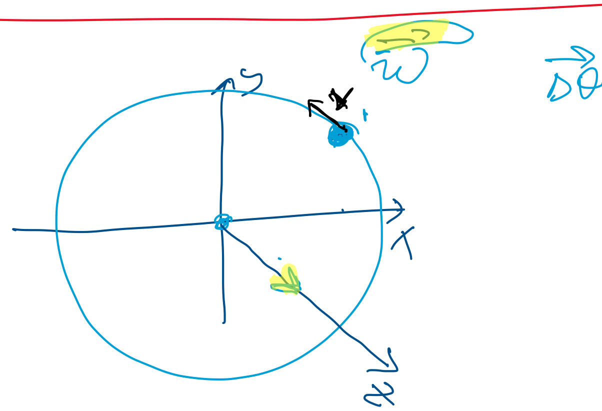
⇒ Rotational Kinematics

Rotational Dynamics

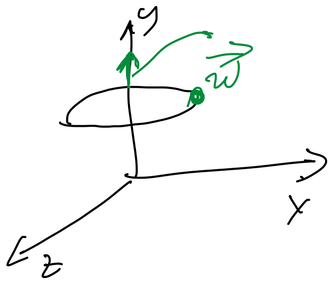
Linear Motion	Rotation
Inertial Mass: M	I – moment of inertia
Force F	$\vec{\tau}$ – rotational Force
1 $\vec{F}_{Net} = 0$ $\quad \vec{V} = const$	(1) $\vec{\tau}^{ext} = 0$ $\quad \vec{\omega} = const$
(2) $\vec{F}_{Net} = m\vec{a}$	(2) $\vec{\tau} = I\vec{\alpha}$
(3) $\vec{F}_{1,2} = -\vec{F}_{2,1}$	(3) $\vec{\tau}_{1,2} = -\vec{\tau}_{2,1}$

$\vec{w}$

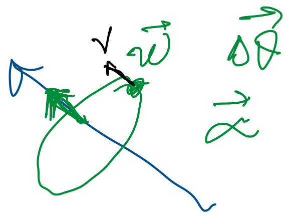


$\vec{w}$
$\vec{\Delta\theta}$
y
x
z
v

v w



y
$\vec{w}$
x
z



v
$\vec{w}$
$\vec{\Delta\theta}$
$\vec{\alpha}$