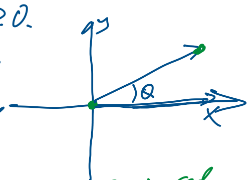
Lecture 14

Thursday, June 1, 2023 8:56 AM

Chapter 9: Rotation of Rigid Bodies

⇒ Kinematics of Rotation

1. Rotation of P.O.
2. Reference Frame
   Axis of Rotation
   Reference direction



3. Position in Space R, θ, ± rad

3' Time t

4. Angular Displacement $\Delta\theta = \theta_f - \theta_i$

4' Time Interval $\Delta t = t_f - t_i$

5. Average Angular Velocity $\omega_{av} = \frac{\Delta\theta}{\Delta t}$, $\frac{rad}{s}$, ±

   Instantaneous $\boxed{\omega = \frac{d\theta}{dt}}$

6. Average Angular Acceleration $\alpha = \frac{\Delta\omega}{\Delta t}$, $\frac{rad}{s^2}$, ±

   Instantaneous $\alpha = \frac{d\omega}{dt}$

Equation of Motion

a) Baby Version $\alpha = 0$

$\omega(t) = \omega_i = const = \omega$

$\theta(t) = \theta_i + \omega t$

b) Junior Version $\alpha = const$

$\omega(t) = \omega_i + \alpha t$

$\theta(t) = \theta_i + \omega_i t + \frac{\alpha t^2}{2}$

$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

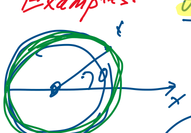
c) Adult Version $\alpha \neq const$

$\omega(t) = \int_i^f \alpha(t)\,dt$

$\theta(t) = \int_i^f \omega(t)\,dt$

Examples: $\theta(t) = At - Bt^2 - Ct^3$

$A =$
$B =$
$C =$



$t = 0$

$$\boxed{f = x^n \qquad \frac{df}{dx} = nx^{n-1}}$$

$\omega(t) = \frac{d\theta(t)}{dt} = A - 2Bt - 3Ct^2$

$\omega(t_s) = 0$

$A - 2Bt_s - 3Ct_s^2 = 0$

$3Ct_s^2 + 2Bt_s - A = 0$

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 3C \qquad b = 2B \qquad c = -A$

$\sqrt{4(B^2+3AC)} \Rightarrow 2\sqrt{B^2+3AC}$

$$t_{1,2} = \frac{-2B \pm \sqrt{4B^2 + 4A\cdot 3C}}{6C} = \frac{-\cancel{2}B \pm \cancel{2}\sqrt{B^2+3AC}}{\cancel{6}C}$$

$$t_{1,2} = \frac{-B \pm \sqrt{B^2+3AC}}{3C} \quad \Bigg| \quad t_s = \frac{-B + \sqrt{B^2+3AC}}{3\cdot C}$$

→ $\alpha(t_s) = ?$

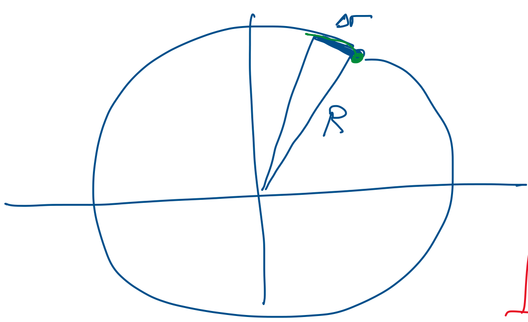
$$\alpha(t) = \frac{d\omega(t)}{dt} = -2B - 6Ct$$

$$\alpha(t_s) = -2B - 6Ct_s$$

— How many Revolutions

$$\theta(t_s) = At_s - Bt_s^2 - Ct_s^3$$

$$N = \frac{\theta(t_s)}{2\pi} = 12.5 \quad \Big| \quad 12$$



$$\boxed{\begin{aligned} \Delta r &= R\Delta\theta \\ V &= R\omega \\ a &= R\alpha \end{aligned}}$$

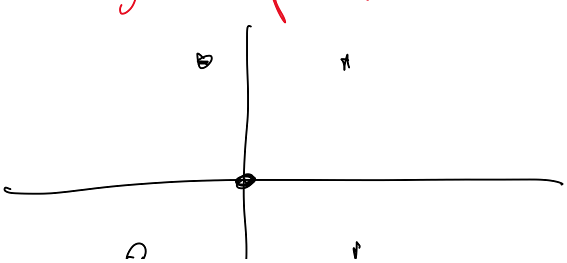
⇒ Kinetic Energy of Rotation

— P.O. $K = \frac{1}{2} I \omega^2$ $\quad \Big| \quad K^{lin} = \frac{1}{2} m V^2$

↳ moment of Inertia

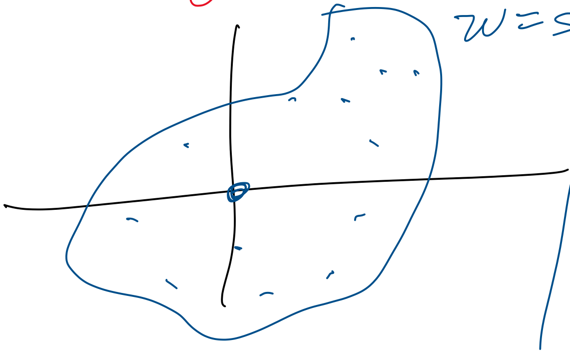
$$\boxed{I = mR^2}$$

— System of Particles $\quad K = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$



$$\boxed{I = \sum_i m_i r_i^2}$$

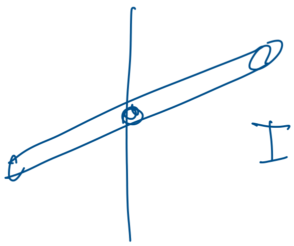
— Rigid Bodies



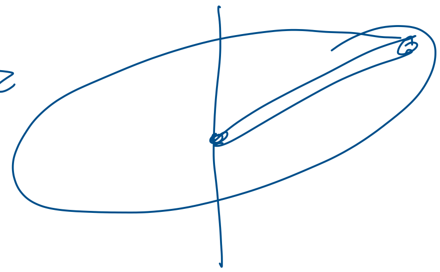
ω = same.

$K = \frac{1}{2} I \cdot \omega^2$

$$\boxed{I = \int r^2 \, dm}$$



$I = \frac{1}{12} ML^2$



$I = \frac{1}{3} ML^2$

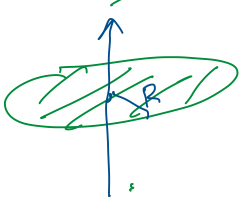
— Solid cylinder

— Hollow Cylinder

— Solid sphere

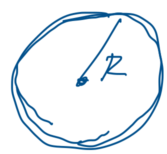
— Hollow Sphere

— Solid Disk



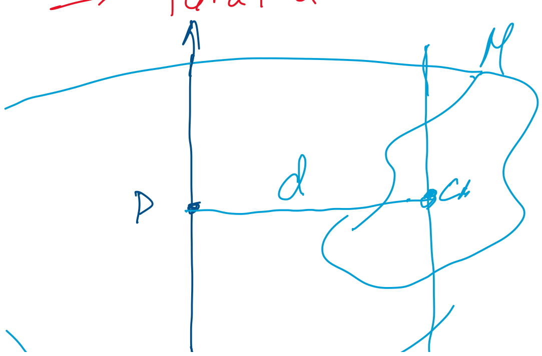
$I = \frac{1}{2} mR^2$

— Hoop



$I = mR^2$

⇒ Parallel-Axis Theorem



I_{CM} = given

$$\boxed{I_P = I_{CM} + Md^2}$$

— Kinematics of rotation

— Dynamics of Rotation

— Linear Motion
Newton

Introduced m

Introduced $\vec{F}$

— 3 Laws of Dynamics

1) $\vec{F}_{net} = 0 \quad \vec{v} = const$

2) $\vec{F}_{Net} = m\vec{a}$

3) $\vec{F}_{1,2} = -\vec{F}_{2,1}$

— Rotational Motion

Introduce I

Angular Force $\vec{\tau}$

— 3 Laws of Rotational Dynamics

1) $\vec{\tau}_{Net} = 0 \quad \vec{\omega} = const$

2) $\vec{\tau}_{Net} = I\vec{\alpha}$

3) $\vec{\tau}_{1,2} = -\vec{\tau}_{2,1}$

$\vec{\tau} = \vec{r} \times \vec{F}$

⇒ Direction for Rotation

$\vec{\Delta\theta}$ $\quad \vec{\omega}, \quad \vec{\alpha}$

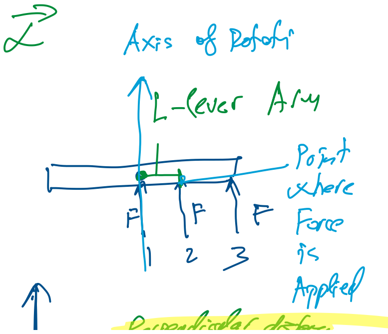
⇒ Angular Force

— Linear Motion $\quad \vec{F}_{Net} = m\vec{a}$

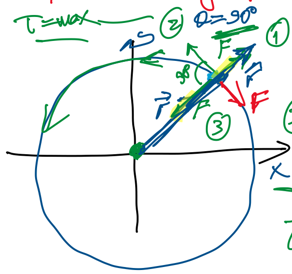
— Rotation $\quad \vec{\tau} = I\vec{\alpha}$

$\tau \sim LF$

Torque



Rotation of Point Object

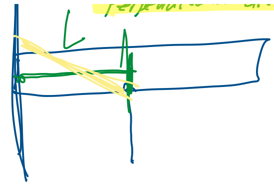


① $= 0$

$\theta = 0$ $\tau = 0$

③ $\theta = 180°$

$\tau = 0$



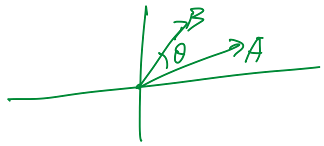
$\tau = |r| F \sin\theta$ $\qquad |A \times B| = AB \sin\theta$

$|\tau| = |r||F| \sin\theta = |\vec{r} \times \vec{F}|$

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

$[\tau] = [r][F][\sin\theta] = [r][F] = m \cdot N$

$\vec{A} \times \vec{B} = \vec{C}$



$I = r^2 m$

$v = \frac{re}{s}$

$K = \frac{1}{2} I \omega^2$

$[K] = [I][\omega^2] =$

$= m^2 \cdot kg \frac{1}{s^2} =$

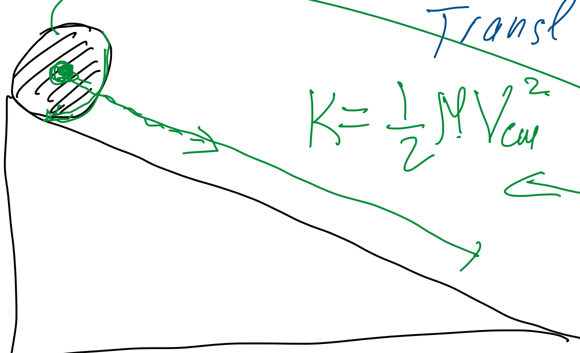
$= m \cdot kg \frac{m}{s^2}$

Linear	Rotation
$\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \cdots = \sum_{i=1}^{N} \vec{F}_i$	$\vec{\tau}_{Net} = \sum_{i=1}^{N} \vec{\tau}_i$

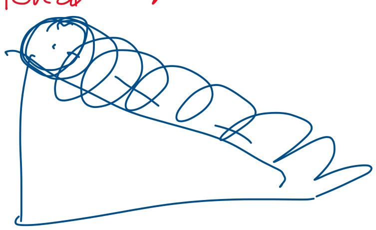
⇒ Linear Motion + Rotation

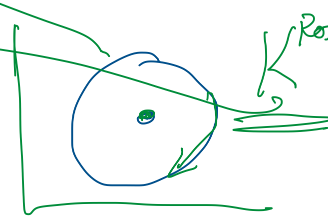
⇒ Combined Linear and Rotational Motion

Translational



$K = \frac{1}{2} M V_{cm}^2$

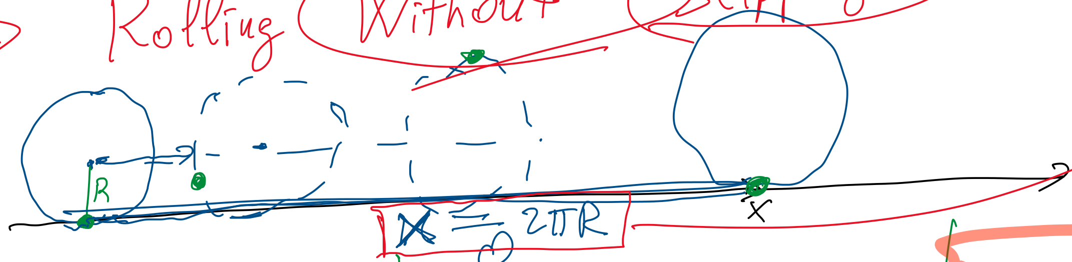




$K^{Rot} = \frac{1}{2} I_{cm} \omega^2$

$$\boxed{K = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} I_{cm} \omega^2}$$

⇒ Rolling Without Slipping



$\theta = 2\pi$

$\omega t = 2\pi$

$$\boxed{\theta = \omega t}$$

$\theta(t) = \theta_i + \omega t$, $\theta_i = 0$

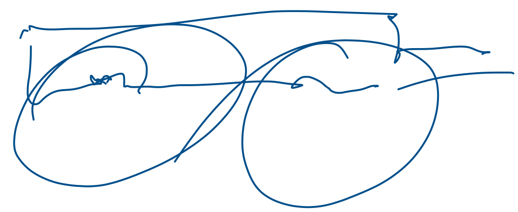
$2\pi = \omega t \quad | \quad t = \frac{2\pi}{\omega}$

$x = v_{cm} \cdot t$

$\cancel{2\pi} R = v_{cm} \cdot \frac{\cancel{2\pi}}{\omega}$

$$\boxed{v_{cm} = R\omega}$$

$$\boxed{v_{cm} = R\omega}$$



$a \neq 0$

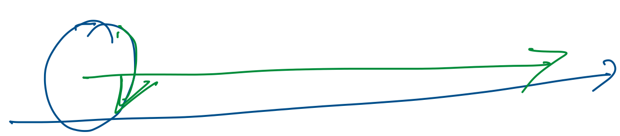
$\frac{d}{dt} v_{cm} = \frac{d}{dt} R\omega = R\frac{d\omega}{dt}$, α

$$\boxed{a_{cm} = R\alpha}$$

⇒ Combined Linear and Rotational Dynamics

$\vec{F}_{net} = \sum \vec{F}_i = M\vec{a}_{cm}$

$\vec{\tau} = \sum \vec{\tau}_i = I\vec{\alpha}$



$L_{net} = \sum v_i - I_{cap}$