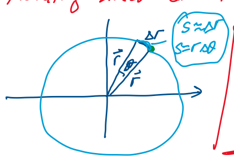
Lecture 15

Monday, June 5, 2023 9:58 AM

Dynamics of Rotational Motion

Linear Motion	Rotation
– Inertial Mass	– Moment of Inertia I (Rotational Motion)
– Force	– Torque $\vec{\tau} = \vec{r} \times \vec{F}$ $\lvert\tau\rvert = rF\sin\theta$ RHR
3 Laws	3 Laws
1) $\vec{F}_{Net} = 0$ $\vec{v} = const$	1) $\vec{\tau}_{Net} = 0$ $\omega = const$
2) $\vec{F}_{Net} = m\vec{a}$	2) $\vec{\tau}_{Net} = I\vec{\alpha}$
3) $\vec{F}_{1,2} = -\vec{F}_{2,1}$	3) $\vec{\tau}_{1,2} = -\vec{\tau}_{2,1}$

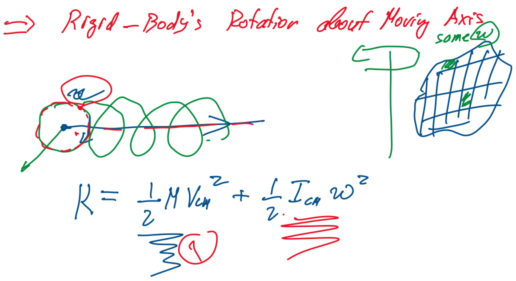
⇒ Relating Linear and Rotational Motion



$$\boxed{\begin{aligned} \Delta r &= r\Delta\theta \\ v &= r\cdot\omega \\ a &= r\cdot\alpha \end{aligned}}$$

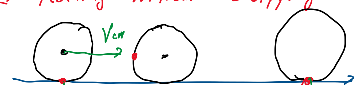
$$\omega = \frac{v}{r}$$

⇒ Rigid-Body's Rotation about Moving Axis



$$K = \frac{1}{2}M V_{cm}^2 + \frac{1}{2} I_{cm}\,\omega^2$$

⇒ Rolling Without Slipping



$L = 2\pi R$

$L = V_{cm}\cdot t$ $\quad L = V_{cm}\cdot \frac{2\pi}{\omega} = 2\pi R$

$2\pi = \omega t$ / $t = \frac{2\pi}{\omega}$

$$\boxed{V_{cm} = R\omega}$$

$a_{cm} = R\cdot\alpha$

$\omega = \frac{V}{}$

⇒ Combined Translation and Rotation (Linear) Dynamics

$\vec{F}_{Net} = \sum_{i=1}^{N} \vec{F}_i$

$\vec{\tau}_{Net} = \sum_{i=1}^{N} \vec{\tau}_i$

$\vec{F}_{Net} = M_{cm}\cdot\vec{a}$

$\vec{\tau}_{Net} = I_{cm}\vec{\alpha}$

$$\boxed{a_{cm} = R\,\alpha_{cm}}$$

⇒ Rotational Work

Linear Motion	Rotational Motion
$\vec{F} = const$	$\vec{\tau} = const$
$W = \vec{F}\cdot\Delta\vec{r}$	$W_{Rot} = \vec{\tau}\cdot\Delta\vec{\theta}$
$\lvert W\rvert = \lvert F\rvert\lvert\Delta r\rvert\cos\theta$	$\lvert W_{Rot}\rvert = \lvert\tau\rvert\lvert\Delta\theta\rvert\cos\theta$
$\vec{F} \neq const$	
$W = \int_1^2 \vec{F}\cdot d\vec{r}$	$W = \int_1^2 \vec{\tau}\cdot d\theta$

Work-Kinetic Energy Theorem

Linear

$W = K_f - K_i$

$K = \frac{1}{2} m v^2$

Rotational

$W = K_f^{Rot} - K_i^{Rot}$

$K^{Rot} = \frac{1}{2} I \omega^2$

Concept of Momentum

Linear Momentum

$\vec{P} = m\vec{v}$

Newton's Second Law

$\boxed{\vec{F} = \frac{d\vec{P}}{dt}} = \frac{dm\vec{v}}{dt} = m\frac{d\vec{v}}{dt}$

$\vec{F} = m\vec{a}$

Angular Momentum

$\vec{L} = I \cdot \vec{\omega}$

$\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$

Point Object

$I = mr^2 \quad \omega = \frac{v}{r}$

r - radius

$|L| = I|\omega|$

$|L| = mr^2 \cdot \frac{v}{r} = r \cdot mv = rP$

$\boxed{\vec{L} = \vec{r} \times \vec{P}}$

Prove

$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$

$\boxed{\frac{d(\vec{r} \times \vec{P})}{dt}} = \frac{d\vec{r}}{dt} \times \vec{P} + \vec{r} \times \frac{d\vec{P}}{dt} = //$

$\frac{d\,ab}{dx} = \frac{da}{dx} \cdot b + a\frac{db}{dx}$

$|\vec{A} \times \vec{B}| = |A||B|\sin\theta$

$// = \vec{v} \times \vec{P} + \vec{r} \times \vec{F} =$

$\vec{v} \times m\vec{v} + \underbrace{\vec{r} \times \vec{F}}_{\vec{\tau}} = m\,\underbrace{\vec{v} \times \vec{v}}_{|v||v|\sin 0} + \vec{\tau}$

$\frac{d\vec{L}}{dt} = \vec{\tau}$

$\boxed{\vec{L} = \vec{r} \times \vec{P}}$

$\boxed{\vec{L} = I\vec{\omega}}$

Elementary Particles

Matter

Electron

e

ν

spin $= \frac{\hbar}{2}$

Quarks

u, d

$\frac{\hbar}{2}$ $\frac{\hbar}{2}$

Interaction Particle

Photon γ = spin $\hbar$

W, Z] Bosons — spin $\hbar$

g — spin $\hbar$

gluons

I

II

III

e — mass

e — charge

e — $s = \frac{h}{4\pi} = \frac{1}{2}\hbar$

$\hbar = \frac{h}{2\pi}$

No 2 Fermions can be at the same state

Pauli Exclusion Principle

particles

half integer $\hbar$ — Fermions

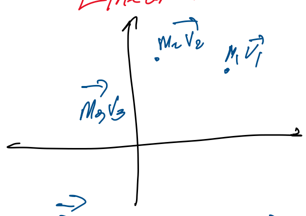
Integer $\hbar$ — Bosons

No Exclusion Principle

Momentum Conservation

⇒ System of Particles

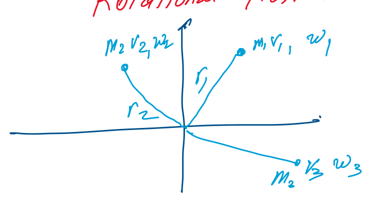
Linear Motion



$$\vec{P}_{TOT} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$
$$= \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

$$\boxed{\vec{P}_{TOT} = \sum_{i=1}^{N} \vec{P}_i}$$

Rotational Motion



Total Angular Momentum

$$\vec{L}_{Tot} = \underbrace{I_1\vec{w}_1}_{\vec{L}_1} + \underbrace{I_2\vec{w}_2}_{\vec{L}_2} + \underbrace{I_3\vec{w}_3}_{\vec{L}_3}$$

$$I = mr^2$$

$$\vec{L}_{TOT} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3$$

$$\boxed{\vec{L}_{TOT} = \sum_{i=1}^{N} \vec{L}_i}$$

Conservation of Momentum

Linear

$$\vec{F}^{ext}_{Net} = 0$$

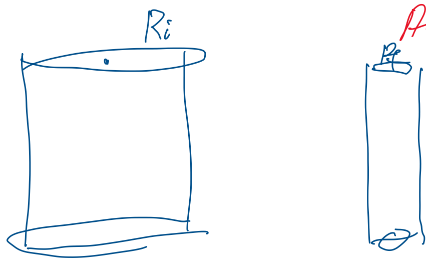
$$\vec{P}_{TOT} = const$$

Rotational Motion

$$\vec{\tau}^{ext}_{Net} = 0$$

$$\boxed{\vec{L}_{TOT} = const}$$

Conservation of Total Angular Momentum



$L_i = I_i \omega_i$ $\qquad$ $L_f = I_f \omega_f$

$I_i = \frac{1}{2} M R_i^2$ $\qquad$ $I_f = \frac{1}{2} M R_f^2$

$L_i = \frac{1}{2} M R_i^2 \omega_i$ $\qquad$ $L_f = \frac{1}{2} M R_f^2 \omega_f$

$$L_i = L_f$$

$$\cancel{\frac{1}{2} M} R_i^2 \omega_i = \cancel{\frac{1}{2} M} R_f^2 \omega_f$$

$$R_i^2 \omega_i = R_f^2 \omega_f$$

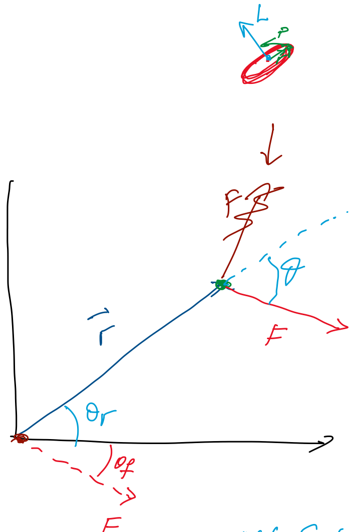
$$\omega_f = \frac{R_i^2}{R_f^2} \omega_i \qquad R_f = \frac{1}{2} R_i$$

$$\omega_f = 4 \omega_i$$

$\Rightarrow$ Gyroscopes.

$L = \vec{r} \times \vec{p} = I \omega$

$L = $ const



$\vec{F}$ $\quad |F|$ $\quad |\theta_f|$

$\vec{r} = |r|$ $\quad |\theta_r|$

$\vec{\tau} = \vec{r} \times \vec{F}$

$|\tau| = |F| |r| \sin(\theta)$

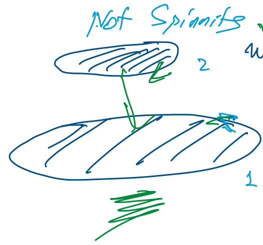
$\theta = \theta_f + \theta_r$

Not Spinning $\omega_2 = 0$

$\omega_2 = 0$ $\quad M_2 R_2$

$I = \frac{1}{2} M R^2$

$\Rightarrow L_i$



L_f $\qquad\qquad\qquad\qquad$ ω_f

$$\vec{L}_i = \vec{L}_1 + \vec{L}_2 = I_1\vec{\omega}_1 + \underbrace{I_2\vec{\omega}_2}_{0} \Rightarrow \boxed{L_i = \frac{1}{2}M_1R_1^2 \cdot \omega_1}$$

$$\vec{L}_f = (I_1+I_2)\vec{\omega_f} = (I_1+I_2)\vec{\omega_f}$$

$$I_1\vec{\omega_f} + I_2\vec{\omega_f}$$

$$L_f = \left(\frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2\right)\omega_f$$

$$L_i = L_f \quad \Big| \quad \frac{1}{2}M_1R_1^2\omega_1 = \left(\frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2\right)\omega_f$$

$$\omega_f = \left(\frac{M_1R_1^2}{M_1R_1^2 + M_2R_2^2}\right)\omega_1$$

- $K_f = \frac{1}{2}(I_1+I_2)\,\omega_f^2$

$$K = \frac{1}{2}I\omega^2$$

- $t_1 = 0$

  $t_2 = 10$ sec

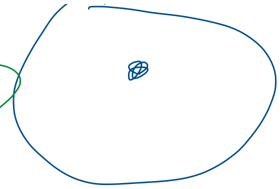
average torque

$$\tau_1 = I_1\frac{\omega_f - \omega_i}{\Delta t}$$

$$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t}$$

$\Rightarrow \quad t_i = 0$

$\omega_i = 20 \, \frac{rad}{s}$



$\alpha_i = 34 \, \frac{rad}{s^2}$

$t_2 = 2 \, sec$ — $\omega_2 (t=2)$

$\theta_2 = 400 \, rad$

$\alpha_2 =$

θ_{TOT}

$\theta_{TOT} = \theta_1 + \theta_2$

before switched off

after it was switched off

$t = t_2$

$\theta_1 = \theta_i + \omega_i t + \frac{\alpha_i t^2}{2}$ ($\theta_i = 0$)

$x = x_i + v_i t + \frac{a_i t^2}{2}$

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what is α_2 ?

$$\boxed{\omega_f^2 - \omega_2^2 = 2 \theta_2 \alpha_2}$$

$v_f^2 - v_i^2 = 2 \Delta x \cdot a$

$$\boxed{\alpha_2} = \frac{\omega_f^2 - \omega_2^2}{2\theta_2} = \frac{-\omega_2^2}{2\theta_2}$$

$\omega_f = 0$

$\omega_2 = \omega_i + \alpha_i \cdot t_2$

– How long it takes to stop

t_2 $\quad \omega_2$

$\omega_f = \omega_2 + \alpha_2 t_s$

$0 \qquad \omega_2 + \alpha_2 t_s = 0$

$$\boxed{t_s} = -\frac{\omega_2}{\alpha_2} > 0$$

$t = t_s + t_2$