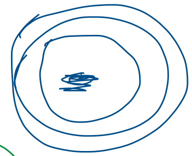
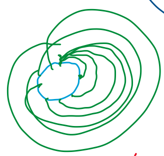
Lecture 16

Tuesday, June 6, 2023 10:03 AM

Aristotle: Heaven, Earth
4-3 c. BC



Galileo?



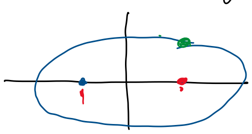
under const acceleration $g = 9.81 \frac{m}{s^2}$?

⇒ Johannes Kepler 1571–1630

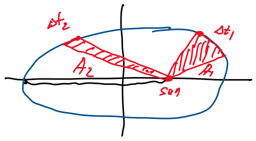
Tycho Brahe: Prague

Kepler's Laws $\boxed{1603\text{–}1619}$

Law 1. All planets move in elliptical orbits wrt sun at one of focus



Law 2 A line joining any planet to the sun sweeps out equal areas in equal times



$\Delta t_1 = \Delta t_2$

$A_1 = A_2$

Law 3 r ⟶ semimajor axis ⟶ mean distance to sun

T — full revolution — Period

$\boxed{T^2 = C r^3}$ ⟹ same for all planets

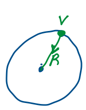
Gravity

⇒ Newton 1687

what Newton knew

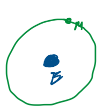
— $\vec{F} = m\vec{a}$

— Centripetal Acceleration



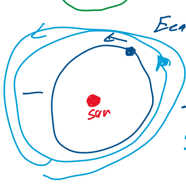
$a_c = \frac{v^2}{R}$

—



F — repels
attracts

Earth attracts Moon



Sun Attracts Earth
Sun Attracts All planets

— 1665 – Outbreak of Bubonic Plague England   Covid 1665

└→ childhood Home

└— apple

Earth Attracts Moon } — same origin ?
Earth Attracts Apple }
} — Same origin
Sun attracts the Planets

Universal Gravity
Gravitational Attraction

⇒ Mathematical Formula

What was known :

$g = 9.81 \frac{m}{s^2}$

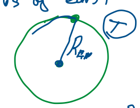
$R_E = 6400\,km$ radius of earth

$R_{EM} = 60 R_E$

$T_M = 27.32$ day

R_{ES} = known

$T_E = 365$ day



$V_M = \frac{2\pi R_{EM}}{T} = 1.022 \frac{km}{s}$

$V_{E,S} = \frac{2\pi R_{ES}}{T_E} = 30 \frac{km}{s}$

Calculate

$a_{cp}^{M,E} = \frac{V_M^2}{R_{EM}}$

$F_{E,M} = M_{Moon}\, a_{cp}^{M,E}$

$|F_{E,M}| \sim M_{moon}$

$|F_{M,E}| \sim M_{Earth}$

$F_{EM} \sim M_{Moon} \cdot M_{Earth}$

Sun $a_{cp}^{E,S} = \frac{V_{ES}^2}{R_{ES}}$

$F_{S,E} = M_{Earth} \cdot a_{cp}^{E,S}$

$$\boxed{F_{ES} = G \frac{M_{sun} \cdot M_{Earth}}{r^n}}$$

!

$n = ?$

On Kepler's Law

⇒ Kepler's 3rd Law

$T^2 = C \cdot r^3$

$F = G \frac{m_S m_P}{r^n}$ $n = 2$,

$$\boxed{a_{cp} = \frac{V_P^2}{r}}$$

$F_{S,P} = m_P \cdot a_{cp}$

$\frac{G\, m_S^G m_P^G}{r^n} = m_P^I\, a_{cp}$

$$\boxed{T = \frac{2\pi r}{V_P}}$$

$\frac{G\, m_S^G m_P^G}{r^n} = m_P^I \frac{V_P^2}{r}$

$V_P = \frac{2\pi r}{T}$

$T^2 \frac{G\, m_S^G m_P^G}{r^n} = \frac{m_P^I (2\pi)^2 r^{\cancel{2}}}{\cancel{r} \cdot T^2} \cdot T^2$

Solve for T^2

r^{n+1}

$$T^2 = \frac{m_P^I 4\pi^2\, r \cdot r^n}{G\, M_S^G\, m_P^G} = \frac{m_P^I 4\pi^2}{G\, m_S^G m_P^G} r^{n+1}$$

$$T^2 = C r^3$$

$$\boxed{n = 2}$$

C is same for all planets

$$C = \frac{m_P^I 4\pi^2}{G\, m_S^G m_P^G}$$

$$\boxed{F_{S,P} = \frac{G M_S \cdot M_P}{r^2}}$$

you need

$$\boxed{m_P^I = m_P^G}$$

→ C ≠ planets

$$\boxed{C = \frac{4\pi^2}{G M_S}}$$

C – you know

G – know

you can calculate

$M_S =$

$$C_{Galaxy} = \frac{4\pi^2}{G\, M_{BH}}$$

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m_1 —— r —— m_2

$$F = \frac{G m_1 m_2}{r^2}$$

Newton's Gravitational Force

q_1 —— r —— q_2

$$F = K \frac{q_1 q_2}{r^2}$$

Coulomb Law

Coulomb const.

Quantum Electrodynamics

q_1 - - - - - - q_2

γ photons

Quantum Theory

$M_{photon} = 0$

m_1 - - - - - - m_2

gravitons

$M_{graviton} = 0$!

Back to Newton

$$\boxed{\vec{F}_{1,2} = -\frac{G m_1 m_2}{r_{12}^2} \hat{r}_{12}}$$

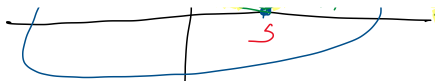
F

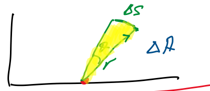
1 $\hat{r}_{12}$ $\quad$ r_{12} 2

$\hat{r}_{12}$

⟹ Kepler's 2nd Law

$\Delta t_1 = \Delta t_2 = \Delta t$ ①





$$\boxed{\Delta A = \frac{1}{2} r \Delta s \sin\theta}$$

Case ①

$\Delta A_1 = \frac{1}{2} r_1 \Delta s_1 \sin\theta_1 =$

$\frac{1}{2} r_1 \underbrace{m V_1}_{m} \Delta t_1 \sin\theta_1 = \frac{1}{2} \overbrace{r_1 P_1 \sin\theta_1}^{|r_1 \times P_1|} \frac{\Delta t_1}{m} = //$

$P = mV$

$|A \times B| = |A||B| \sin\theta$

$$\boxed{\Delta A_1 = \frac{1}{2} |r_1 \times P_1| \frac{\Delta t_1}{m} = \frac{1}{2} |L_1| \frac{\Delta t_1}{m}}$$

$$\boxed{\Delta A_2 = \frac{1}{2} |L_2| \frac{\Delta t_2}{m}}$$

$$\boxed{L = r \times P}$$

Angular Momentum

$\Delta A_1 = \frac{1}{2} |L_1| \frac{\Delta t}{m}$

$\Delta A_2 = \frac{1}{2} |L_2| \frac{\Delta t}{m}$

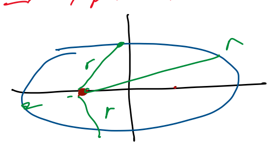
$$\boxed{\Delta A_1 = \Delta A_2}$$

$$\boxed{\vec{L}_1 = \vec{L}_2}$$

$$\boxed{\vec{L} = \text{Const}}$$

$\vec{\tau}_{ext} = 0$

⇒ Kepler's First Law



$\vec{F}_G = G \frac{M_S M_p}{r^2} \hat{r}$

r = const or changes

$\vec{F}_G$ — changing with $\vec{r}$

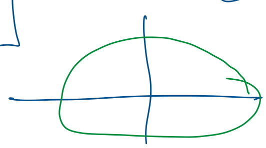
$\vec{F}_g = M_p \vec{a}_p$ → changing

$v(t) = \int_1^2 a(t)\, dt$

$r(t) = \int_1^2 v(t)\, dt$

$r(t) =$ 

$$\boxed{\text{Elipse}}$$



- what is g
- Escape velocity
- Black holes: