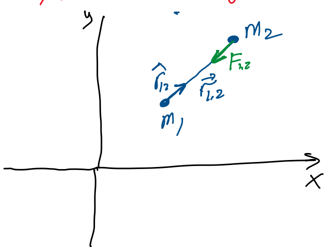
Lecture 17

Wednesday, June 7, 2023 9:53 AM

Final Formula for Gravitational Force



$$\vec{F}_{1,2} = -G\frac{m_1 m_2}{r_{1,2}^2}\hat{r}_{12} = -\frac{G m_1 m_2 \vec{r}_{12}}{r_{12}^3}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$

$$\left|\frac{\vec{r}_{12}}{|\vec{r}_{12}|}\right| = \frac{|\vec{r}_{12}|}{|\vec{r}_{12}|} = 1$$

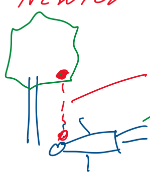
m_1 m_2 — gravitational mass

in principle not same as inertial mass

– Fact $\boxed{m^G = m^I}$!

– $G = 6.67 \times 10^{-11} N \frac{m^2}{kg^2}$

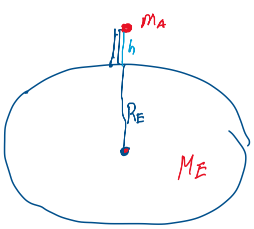
⇒ Newton and Apple



same as above Gravity

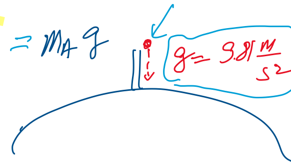
Newtons Second Law

$\boxed{F = ma}$



$$F_{EA} = G\frac{m_A M_E}{(R_E+h)^2}$$

$$F = m_A g$$



$\boxed{g = 9.81\frac{m}{s^2}}$

$$\frac{G \cancel{m_A} M_E}{(R_E+h)^2} = \cancel{m_A} g$$

$$\boxed{g = \frac{G M_E}{(R_E+h)^2}}$$

$h = 3 m$

$R_E = 6400000 m$

$h = 0$

$$\boxed{g = \frac{G M_E}{R_E^2}} = 9.81\frac{m}{s^2}$$

$$M_E = \frac{g R_E^2}{G}$$

$g_{Moon} = 1.6 \frac{m}{s^2}$

R...

M... $= g_{Moon} R_{Moon}^2$

R_{Moon} — T_{Moon} — G

$$\boxed{g = \frac{G M_E}{(R_E+h)^2}}$$

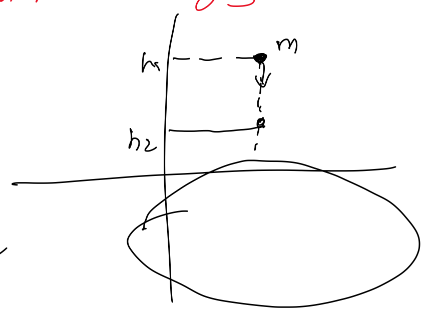
$h = 0$ $\quad 9.81 \frac{m}{s^2}$

$h = 10km$ $\quad g = 9.77 \frac{m}{s^2}$

$h = 40km$ $\quad g = 9.67 \frac{m}{s^2}$

g — what it is

⇒ Gravitational Potential Energy



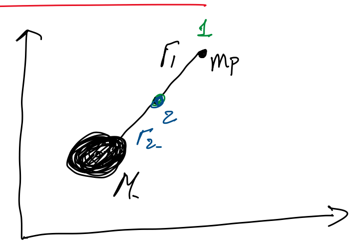
$$W = U_1 - U_2$$

$$W = mgh_1 - mgh_2$$

$$W = F \cdot \Delta y = -mg(h_2 - h_1) = mgh_1 - mgh_2$$

$$U = mgh \qquad F = -mg$$

$$\vec{F} = -\frac{G M_1 M_2}{r_{12}^2} \hat{r}_{12}$$



$$W = \int_1^2 \vec{F} \cdot d\vec{r} = -\int_{r_1}^{r_2} \frac{G M m_p}{r^2} dr = //$$

$$// = -G M m_p \int_{r_1}^{r_2} \frac{dr}{r^2} =$$

$$\boxed{\int \frac{dr}{r^2} = -\frac{1}{r}}$$

$$= G M m_p \frac{1}{r}\Big|_{r_1}^{r_2} = \frac{G M m_p}{r_2} - \frac{G M m_p}{r_1} = U_1 - U_2$$

U_2

$r_2 = R_E + h_2$

$r_1 = R_E + h_1$

$$\boxed{U = -\frac{G M m_p}{r}}$$

True Gravitational Potential Energy

I potential ...

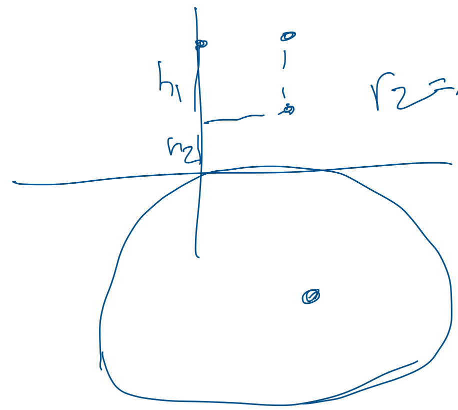
Not $U = mgh$ II

— I $U_g(\text{on Earth}) = -\frac{GMm_p}{R_E}$!

— II $U(\text{on Earth}) = 0$

$r_1 = h_1 + R_E$

$r_2 = h_2 + R_E$



$$W_g = mg(h_1 - h_2)$$

$$W_g = \frac{GMm_p}{R_E + h_2} - \frac{GMm_p}{R_E + h_1}$$

$$\boxed{\frac{1}{R+h} = \frac{1}{R\left(1+\frac{h}{R}\right)} \approx \frac{1}{R}\left(1 - \frac{h}{R}\right)} \quad \frac{1}{R} - \frac{h}{R^2}$$

$h \ll R$

$$W_g = GMm_p\left(\frac{1}{R} - \frac{h_2}{R^2}\right) - GMm_p\left(\frac{1}{R} - \frac{h_1}{R^2}\right)$$

$$W_g = \cancel{GMm_p\frac{1}{R}} - \overbrace{\frac{GMm_p}{R^2}}^{g} h_2 - \cancel{\frac{GMm_p}{R}} + \overbrace{\frac{GMm_p}{R^2}}^{g} \ldots$$

$$\boxed{W_g = mgh_1 - mgh_2}$$

$h \ll R_E$

$U = \frac{GMm_p}{\ldots}$ — on Earth $U = -\frac{G\ldots}{\ldots}$

$U_g = -\frac{GM}{r}$

$E_2 = U + \frac{1}{2} m_p v_f^2 = K_2$

$r \to \infty$

⇒ Escape Velocity

1 m_p, V $K_1 = \frac{m_p V^2}{2}$ | $E_1 = U + K = -\frac{GMm_p}{R_E} + \frac{1}{2} m_p V^2$

$E_1 = E_2$ | $-\frac{GMm_p}{R_E} + \frac{1}{2} m_p V^2 = \frac{1}{2} m_p V_f^2$ $V_f = 0$

2 ∞

1

$V_E = ?$

$$-\frac{GMm_p}{R_E} + \frac{1}{2} m_p V_E^2 = 0$$

$\frac{1}{2} \cancel{m_p} V_E^2 = \frac{GM\cancel{m_p}}{R_E}$ | $V_E^2 = \frac{2GM}{R_E}$

$V_E = \sqrt{\frac{2GM}{R}} = \sqrt{2gR_E}$ Earth

$V_E = \sqrt{\frac{2GM \, R_E}{R_E \, R_E}} = \sqrt{2gR_E}$ g

$V_E = \sqrt{2gR_E} = 11.2 \frac{km}{s} = 25000 \frac{mi}{h}$

$9.81 \frac{m}{s^2}$ 6400000 m

$$V_E^{sun} = \sqrt{\frac{2\,G\,M_{sun}}{R_{sun}}} = 618 \frac{km}{s} = 1381600 \frac{mi}{h}$$

$$V_E^{Moon} = \sqrt{\frac{2G\,M_{moon}}{R_{moon}}} = 2.38 \frac{km}{s} = 5320 \frac{mi}{h}$$

Escape Planetary System

$$V_E^{P.S} = \sqrt{\frac{2\,G\,(M_{sun} + M_{planets})}{R_{P.S}}} = 42. \frac{km}{s}$$

$\Rightarrow$ Voyager Missions — 1977 October

Mounted on Voyager 1 and Voyager 2, twin probes launched in 1977, the two copies of the record would serve as time capsules and transmit much more information about life on Earth should extraterrestrials find it.

$\Rightarrow$ Black Holes

$$V_E = \sqrt{\frac{2GM_s}{R_s}}$$

$M_s,\ R_s$

$V_E > C - 186000\,\frac{mi}{s} = 3\times10^8\,\frac{m}{s}$

$\Rightarrow$ M_{sun} – R_{sun} if to become a B.H

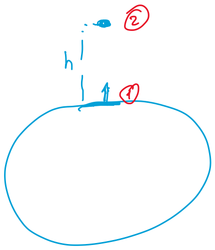
$V_E = C$

$$\boxed{C = \sqrt{\frac{2GM_{sun}}{R_{BH}}}} \Rightarrow C^2 = \frac{2GM_s}{R_{B.H}}$$

$$\boxed{R_{B.H} = \frac{2GM_s}{C^2}}$$

$R_s = 3\,km$

Example



$h = 200\,km$
$m = \ldots$
$V_f = \ldots$

V_i

$$U_g = -\frac{GM_E m}{r}$$

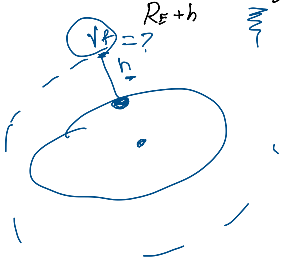
$E^{(1)} = U_1 + K_1$
$\|$
$E^{(2)} = U_2 + K_2$

$V_i = ?$

$$E^{(1)} = -\frac{GM_E m_s}{R_E} + \frac{1}{2} m_s V_i^2$$
$\|$
$$E^{(2)} = -\frac{GM_E m_s}{R_E + h} + \frac{1}{2} m_s V_f^2$$

$$-\frac{GM_E m_s}{R_E} + \frac{1}{2} m_s V_i^2 = -\frac{GM_E m_s}{R_E + h} + \frac{1}{2} m_s V_f^2$$

$V_f = ?$



ω

ω_{min}

$24h \rightarrow \Delta\theta = 2\pi$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{24\cdot60\cdot60} = (N_0)\,\frac{rad}{s}$$

$V_f =$

$V = \omega\cdot r$, $\quad V_f = \omega\cdot(R_E + h)$

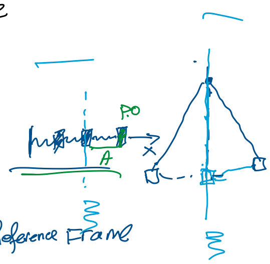
1. Linear Motion → 1d, 2d } Kinematics, Dynamics
2. Rotational Motion 2d } Kinematics, Dynamics
3. Periodic Motion
4. Wave Motion

Periodic Motion

– Motion that repeats itself after Sometime

– Oscillations

⇒ Describe Oscillations



Spring

1) Equilibrium Point (E.P.) Reference Frame

2) Position of (P.O) around E.P., $X(t)$, m, ±

3) Changing the position during the time
Displacement

4) Velocity of P.O. $V(t) = \frac{dX(t)}{dt}$, $\frac{m}{s}$ ±
Instantaneous Velocity

5) Instantaneous acceleration $a(t) = \frac{dV(t)}{dt}$

Equation of Motion
$a(t) \neq$ const (Adult Version)

$$V(t) = \int_0^t a(t)\,dt$$

$$X(t) = \int_0^t V(t)\,dt$$

6) Amplitude A, m ±

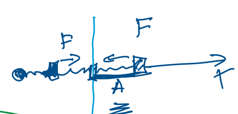
7) Period T, s ±

8) Frequency $\frac{1}{T} = f$, $[f] = \frac{1}{s} = 1$ Hertz
Heinrich Hertz (1857 – 1894)

8') Angular Frequence
$\omega = 2\pi f$ $[\omega] = \frac{rad}{s}$

⇒ Dynamics

Force ⟶ Restoring Force



$$F_x = -KX(t)$$

2) $F_x = m \cdot a_x$ $F_x(t) = m\,a_x(t)$ ⟹ $-KX(t) = m\,a_x(t)$ $a_x(t) = -\frac{K}{m}X(t)$

$$a_x(t) = \frac{dV(t)}{dt} = \frac{d^2X(t)}{dt^2}$$ – Second Derivative of $X(t)$

$V = \frac{dx}{dt}$

$$-KX(t) = m\frac{d^2X(t)}{dt^2}$$

$$m\frac{d^2X(t)}{dt^2} + KX(t) = 0$$

Equation of Simple Harmonic Motion

→ Solution

$$X(t) = A\cos(Bt + C)$$

A, B, C – are Constants ?

$f(t) = \cos(Bt+C)$

$$\frac{df(t)}{dt} = -B\sin(Bt+C)$$

$$V(t) = \frac{dX(t)}{dt} = A\frac{d\cos(Bt+C)}{dt} = -AB\sin(Bt+C)$$

$f_2(t) = \sin(Bt+C)$

$$\frac{d^2X(t)}{dt^2} = \frac{dV(t)}{dt} = -AB\frac{d\sin(Bt+C)}{dt} = -AB^2\cos(Bt+C)$$ $\frac{df_2(t)}{dt} = B\cos(Bt+C)$

$= -B^2 A\cos(Bt+c)$

$- B^2 X(t)$

$$\boxed{\frac{d^2X(t)}{dt^2} = -B^2 X(t)}$$

$$-mB^2X(t) + KX(t) = 0 \quad \Big| \quad \frac{mB^2\cancel{X(t)}}{\cancel{X(t)}} = \frac{K\cancel{X(t)}}{\cancel{X(t)}}$$

$$-mB^2 + K = 0$$

$$mB^2 = K$$

$$B^2 = \frac{K}{m}; \quad \boxed{B = \sqrt{\frac{K}{m}}}$$

$$X(t) = A\cos(Bt + C) \qquad \boxed{B = \sqrt{\frac{K}{m}}}$$

$$\boxed{A = \text{. Amplitude}}$$

$$X^{max} = A$$

$$\rightarrow X(0) = A\cos(C)$$

$$||$$

$$\rightarrow X(T) = A\cos(BT + C)$$

$$\cancel{A}\cos C = \cancel{A}\cos(BT+C)$$

$$\cos(C) = \cos(BT + C)$$

$$\boxed{BT = 2\pi}$$

$$\cos(C) = \cos(2\pi + C) = \cos(C)$$

$$BT = 2\pi$$

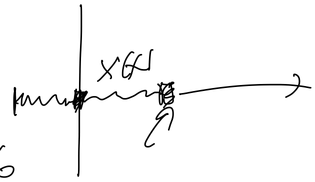
$$\boxed{B = \frac{2\pi \cdot 1}{T} = 2\pi \cdot f = \omega}$$

$$\cos(2\pi + \theta) = \cos\theta$$

$$V(t) = A\cos(\omega t + C)$$

$$\boxed{\omega = B = \sqrt{\frac{K}{m}}}$$

$x(0) = A\cos(c)$



c — defines $x(0) = x_0$

→ $\boxed{\text{phase}}$ → φ

$2\pi f$

$\omega = \sqrt{\frac{k}{m}}$

A — amplitude

φ — phase

(1) $\boxed{x(t) = A\cos(\omega t + \varphi)}$

$v(t) = \frac{dx(t)}{dt} = \frac{d\,A\cos(\omega t+\varphi)}{dt} = -\omega A\sin(\omega t+\varphi)$

(2) $\boxed{v(t) = -\omega A\sin(\omega t+\varphi)}$ ← Equation of Motion

$\boxed{\begin{aligned} x_i &= A\cos\varphi \\ v_i &= -\omega A\sin\varphi \end{aligned}}$

$\frac{v_i}{x_i} = \frac{-\omega A\sin\varphi}{A\cos\varphi} = -\omega\tan\varphi$

$\boxed{\varphi = \tan^{-1}\left(-\frac{v_i}{\omega x_i}\right)}$

$(x_i)^2 = (A\cos\varphi)^2$

$\left(-\frac{v_i}{\omega}\right)^2 = (A\sin\varphi)^2$

$x_i^2 = A^2\cos^2\varphi$

$+ \quad \frac{v_i^2}{\omega^2} = A^2\sin^2\varphi$

$x_i^2 + \frac{v_i^2}{\omega^2} = A^2\cos^2\varphi + A^2\sin^2\varphi = A^2\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1}$

$A^2 = x_i^2 + \frac{v_i^2}{\omega^2}$

$\boxed{A = \sqrt{x_i^2 + \frac{v_i^2}{\omega^2}}}$