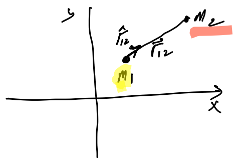
Lecture 18

Thursday, June 8, 2023 9:54 AM

Newton's Universal Gravity



- $\vec{F}_{12} = -G\dfrac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$
- $m_1, m_2 \rightarrow$ Gravitational Mass
- $F = ma \rightarrow$ Inertial Mass

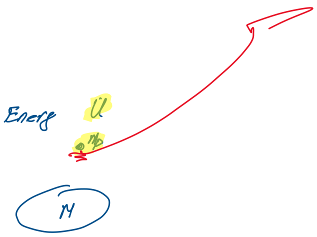
  $m^G \equiv m^I$
- $G = 6.67\times10^{-11}\ \dfrac{N\cdot m^2}{kg^2}$

- $\boxed{g = \dfrac{GM_E}{R_E^2}}$

- $\boxed{g_h = \dfrac{GM_E}{(R_E+h)^2}}$

- Gravitational Potential Energy U

  $\boxed{U = -\dfrac{GMm_p}{r}}$



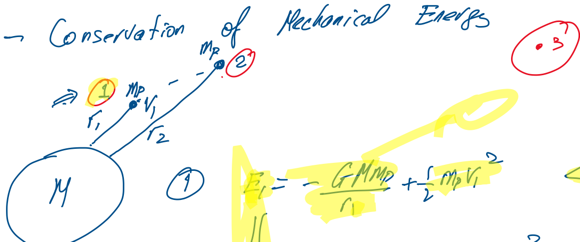
- Total Mechanical Energy Under Gravity

  $$\boxed{E_p = U + K = -\frac{GMm_p}{r} + \frac{1}{2}m_p V_p^2}$$

- Escape Speed

  $$V_E = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

- Conservation of Mechanical Energy



(1) $E_1 = -\dfrac{GMm_p}{r_1} + \dfrac{1}{2}m_p V_1^2 < 0$

$\parallel$

$E_2 = -\dfrac{GMm_p}{r_2} + \dfrac{1}{2}m_p V_2^2$

$\parallel$

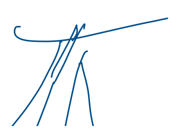
$E_3 = \dfrac{1}{2}m_p V_3^2 \geq 0$!

- Black Holes:

  $V_E = \sqrt{\dfrac{2GM_s}{R_s}}$ $\quad V_E = C$ $\quad \boxed{C = \sqrt{\dfrac{2GM_s}{R_s}}}$

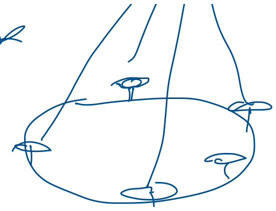
  Given mass

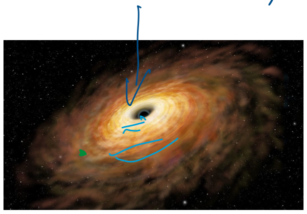
  $R = $ B.H

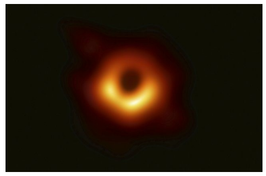


– Supermassive Black Hole in the Center of Milky way

M87, 2019



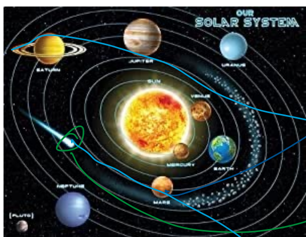




Artist's Picture

⇒ Classification of Orbits & Energy

– Solar System

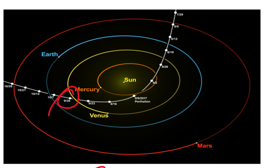


$$E = -\frac{G M_s M_m}{r_{sm}} + \frac{1}{2} M_m V_m^2 < 0$$

$$E = -\frac{G M_s M_o}{r_{so}} + \frac{1}{2} m_o V_o^2 > 0$$

$$E = < \sim 0$$

Oumuamua
(October – 2017

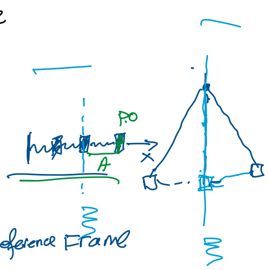


Periodic Motion

– Motion that repeats itself after Sometime

– Oscillations

⇒ Describe Oscillations



Spring

1) Equilibrium Point (E.P) Reference Frame

2) Position of (P.O) around E.P., $X(t)$, m, t

3) Changing the position during the time
Displacement

4) Velocity of P.O. $V(t) = \frac{dX(t)}{dt}$, m/s t
Instantaneous Velocity

5) Instantaneous acceleration $a(t) = \frac{dV(t)}{dt}$

Equation of Motion

$a(t) \neq$ const (Adult Version)

$$V(t) = \int_0^t a(t)\,dt$$

$$X(t) = \int_0^t V(t)\,dt$$

6) Amplitude A, m +

7) Period T, s +

8) Frequency $\frac{1}{T} = f$, $[f] = \frac{1}{s} = 1$ Hertz

Heinrich Hertz (1857 - 1894)

8') Angular Frequence $\omega = 2\pi f$ $\quad [\omega] = \frac{rad}{s}$

⇒ Dynamics

Force → Restoring Force

$$F_x = -KX(t)$$

2) $F_x = m \cdot a_x$ $\quad F_x(t) = m\,a_x(t)$ ⇒ $-KX(t) = m\,a_x(t)$ $\quad a_x(t) = -\frac{K}{m}X(t)$

$$a_x(t) = \frac{dV(t)}{dt} \xleftarrow{V = \frac{dx}{dt}} = \frac{d^2X(t)}{dt^2}$$ — Second Derivative of $X(t)$

$$-KX(t) = m\frac{d^2X(t)}{dt^2} \qquad m\frac{d^2X(t)}{dt^2} + KX(t) = 0$$

Equation of Simple Harmonic Motion

→ Solution

$$X(t) = A\cos(Bt + C)$$

A, B, C - are Constants ?

$f(t) = \cos(Bt+C)$

$$V(t) = \frac{dX(t)}{dt} = A\frac{d\cos(Bt+C)}{dt} = -AB\sin(Bt+C)$$

$$\frac{df(t)}{dt} = -B\sin(Bt+C)$$

$f_2(t) = \sin(Bt+C)$

$$\frac{d^2X(t)}{dt^2} = \frac{dV(t)}{dt} = -AB\frac{d\sin(Bt+C)}{dt} = -AB\cos(Bt+C)$$

$$\frac{df_2(t)}{dt} = B\cos(Bt+C)$$

$$= -B^2 A\cos(Bt+C)$$

$$-B^2X(t)$$

$$\frac{d^2X(t)}{dt^2} = -B^2X(t)$$

$$-mB^2X(t) + KX(t) = 0 \quad \Big| \quad \frac{mB^2\cancel{X(t)}}{X(t)} = \frac{K\cancel{X(t)}}{X(t)}$$

$$-mB^2 + K = 0$$

$$mB^2 = K$$

$B^2 = \frac{K}{m}$ $\quad B = \sqrt{\frac{K}{\ }}$

$B^2 = \frac{k}{m}$; $\boxed{B = \sqrt{\frac{k}{m}}}$

$x(t) = A\cos(Bt + C)$ $\boxed{B = \sqrt{\frac{k}{m}}}$

$\boxed{A = \text{Amplitude}}$

$x^{max} = A$

$\rightarrow x(0) = A\cos(C)$

$||$

$\rightarrow x(T) = A\cos(BT + C)$

$A\cos C = A\cos(BT + C)$

$\cos(C) = \cos(BT + C)$

$\boxed{BT = 2\pi}$

$\cos(C) = \cos(2\pi + C) = \cos(C)$

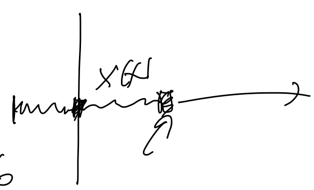
$BT = 2\pi$

$\boxed{B = \frac{2\pi}{T} = 2\pi \cdot f = \omega}$ f

$\cos(2\pi + \theta) = \cos\theta$

$x(t) = A\cos(\omega t + C)$ $\boxed{\omega = B = \sqrt{\frac{k}{m}}}$

$x(0) = A\cos(C)$



C – defines $x(0) = x_0$

$\rightarrow$ $\boxed{\text{phase}} \rightarrow \varphi$

$\boxed{x(t) = A\cos(\omega t + \varphi)}$

$2\pi f$

$||$

$\omega = \sqrt{\frac{k}{m}}$

A – amplitude

φ – phase

$V(t) = \frac{dx(t)}{dt} = \frac{d\,A\cos(\omega t+\varphi)}{dt} = -\omega A\sin(\omega t+\varphi)$

1) $\boxed{V(t) = -\omega A\sin(\omega t+\varphi)}$

Equation of Motion

$\boxed{\begin{aligned} X_i &= A\cos\varphi \\ V_i &= -\omega A\sin\varphi \end{aligned}}$

$\frac{V_i}{X_i} = \frac{-\omega A\sin\varphi}{A\cos\varphi} = -\omega\tan\varphi$

$\boxed{\varphi = \tan^{-1}\left(-\frac{V_i}{\omega X_i}\right)}$

$(X_i)^2 = (A\cos\varphi)^2$

$\left(-\frac{V_i}{\omega}\right)^2 = (A\sin\varphi)^2$

$X_i^2 = A^2\cos^2\varphi$

$+$

$\frac{V_i^2}{\omega^2} = A^2\sin^2\varphi$

$X_i^2 + \frac{V_i^2}{\omega^2} = A^2\cos^2\varphi + A^2\sin^2\varphi = A^2\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1}$

$A^2 = X_i^2 + \frac{V_i^2}{\omega^2}$

$\boxed{A = \sqrt{X_i^2 + \frac{V_i^2}{\omega^2}}}$