Lecture 19

Monday, June 12, 2023 10:15 AM

Physics of Motion

1. Linear Motion $\xrightarrow{1d}$ Kinematics, $\xrightarrow{2d}$ Dynamics

2. Rotational Motion { Kinematics, Dynamics }

3. Periodic Motion/Oscillations

4. Wave Motion

Periodic Motion

– Motion that repeats itself

– Oscillations

⇒ Describing Oscillations   Spring   Pendulum

maximal displacement

Spring

0. Point object P.O.

1. Equilibrium Point (E.P)
   Reference Frame 1d

2) Position of P.O. around E.P. x, m $\underline{+}$

2)' Time

3) Displacement change of position during the time $x(t)$ m, $\underline{+}$

4) Instantaneous Velocity $v(t) = \frac{dx(t)}{dt}$

5) Instantaneous Acceleration $a(t) = \frac{dv(t)}{dt}$

6) Maximal Displacement A – amplitude +, m

7) Period T, s +

8) Frequence $f = \frac{1}{T}$, $[f] = \frac{1}{s} = 1$ Hertz

9) Angular Frequence $\omega = 2\pi f$ $[\omega] = \frac{rad}{s}$

Periodic Motion

Equations of Motion | Spring

Equation of ... Spring Force

$\Rightarrow$ Adult Version of Motion

$$F = ma \quad = -kx = ma$$

$a \neq$ const $\quad a = -\frac{kx}{m}$

$$\boxed{F = -kx}$$

Const.

$\rightarrow$ displacement

$\rightarrow$ Restoring Force

$$V(t) = \int_0^t a(t)\,dt \qquad V(t) = ?$$

$$X(t) = \int_0^t V(t)\,dt \qquad X(t) = ?$$

$\Rightarrow$ Newton's Second Law

$$F = ma \Rightarrow -kx = m\frac{d^2x}{dt^2}$$

1. $v = \frac{dx}{dt}$

$$m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$

$$-kx(t) = m\frac{d^2x}{dt^2} \Rightarrow \boxed{m\frac{dx^2}{dt^2} + kx(t) = 0}$$

Solution

Equation of Symple Harmonic Motion

$$x(t) = A\cos(Bt+C)$$

$$f(t) = \cos(Bt+C)$$

$$\frac{df(t)}{dt} = -B\sin(Bt+C)$$

$$\frac{d^2f(t)}{dt^2} = -B\frac{d\sin(Bt+C)}{dt} = -B\left(B\cos(Bt+C)\right)$$

$B\cos(Bt+C)$

$$\Longrightarrow -B^2\cos(Bt+C)$$

$$m\frac{d^2 \cancel{A}\cos(Bt+C)}{dt^2} + K\cancel{A}\cos(Bt+C) = 0$$

$$m\frac{d^2\cos(Bt+C)}{dt^2} + K\cos(Bt+C) = 0$$

$$-mB^2\cancel{\cos(Bt+C)} + K\cancel{\cos(Bt+C)} = 0$$

$$-mB^2 + K = 0 \Longrightarrow B^2 = \frac{K}{m}, \quad \boxed{B = \sqrt{\frac{K}{m}}}$$

1

$$\boxed{x(t) = A\cos(Bt+C)} + \boxed{B = \sqrt{\frac{K}{m}}}$$

$X_{max} = A =$ amplitude

$t=0$

$x(0) = A\cos(C)$

$x(T) = A\cos(BT + C)$

$A\cos(C) = A\cos(BT+C)$

$\cos C = \cos(BT+C)$

$\boxed{BT = 2\pi}$

$B = \sqrt{\frac{k}{m}} = \omega$

$B = \frac{2\pi}{T} = 2\pi f = \omega$

$\frac{1}{T} = f$

$t=0$

$x(0) = A\cos C$

→ **Phase factor** $= \varphi$

Summarizing

$\boxed{x(t) = A\cos(\omega t + \varphi)}$ $\boxed{\omega = \sqrt{\frac{k}{m}}}$ → Simple Harmonic Motion

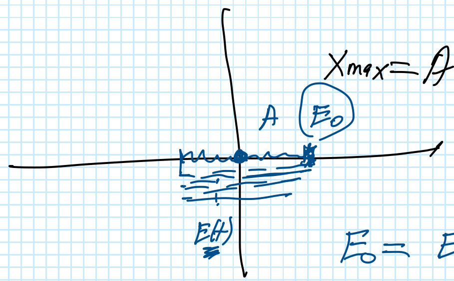
$\boxed{v(t)} = \frac{dx(t)}{dt} = \frac{A\, d\cos(\omega t + \varphi)}{dt} = \boxed{-\omega A\sin(\omega t + \varphi)}$

⟹ Energy of Periodic Motion on Spring

$U_s = \frac{1}{2}kx$

$$E_{TOT} = K + U_s$$

$$E_0 = 0 + \frac{1}{2}kA^2 = \frac{1}{2}kA^2$$



$E_0 = E(t)$

$$E_{Tot}(t) = \frac{1}{2}mv^2(t) + \frac{1}{2}kx^2(t)$$

$$E_{TOT}(t) = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t+\varphi) + \frac{1}{2}kA^2\cos^2(\omega t+\varphi) =$$

$$E_{TOT}(t) = \frac{1}{2}\cancel{m}\left(\frac{k}{\cancel{m}}\right)A^2\sin^2(\omega t+\varphi) + \frac{1}{2}kA^2\cos^2(\omega t+\varphi)$$

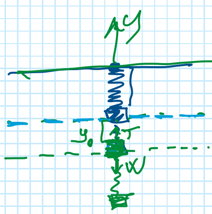
$$= \frac{1}{2}kA^2\sin^2(\omega t+\varphi) + \frac{1}{2}kA^2\cos^2(\omega t+\varphi)$$

$$\frac{1}{2}kA^2\Big(\underbrace{\sin^2(\omega t+\varphi) + \cos^2(\omega t+\varphi)}_{1}\Big) = \frac{1}{2}kA^2$$

$$\boxed{E_0 = \frac{1}{2}kA^2}$$

$$\boxed{E_{Tot}(t) = \frac{1}{2}kA^2}$$

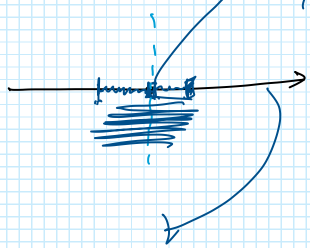
Vertical Spring



Old E.P

New E.P

E.P. / $\vec{F}_{net} = 0$



$$\vec{T} + \vec{W} = 0$$

$$ky_0 - mg = 0$$

$$ky = mg$$

$$\boxed{y_0 = \frac{mg}{k}}$$ New E.P.

$$\boxed{y' = y - y_0}$$

$$y'(t) = y(t) - y_0$$

$$m\frac{d^2y'(t)}{dt^2} = -k\,y'(t)$$

$$y'(t) = A\cos(\omega t + \varphi)$$

$$v(t) = -\omega A \sin(\omega t + \varphi)$$

⇒ Pendulum

⇒ Rotational Simple Harmonic Motion

1. E.P. $\theta = 0$

2. Position of Point Particle $\theta(t)$ rad

3) Angular Velocity $\omega = V_\theta = \frac{d\theta(t)}{dt}$ $\left(\frac{rad}{s}\right)$

4) Angular Acceleration $\alpha(t) = \frac{dV_\theta}{dt} = \frac{d^2\theta}{dt^2}$

5) Amplitude $\theta^{max} = \theta_0$

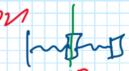
6) Period T, $f = \frac{1}{T}$, $\omega = 2\pi f$

7) Restoring Torque

$\vec{\tau} = -\kappa\theta$ — Torsion Constant

8) $\vec{\tau} = I\vec{\alpha}$

9) $-\kappa\theta(t) = I\frac{d^2\theta(t)}{dt^2}$



Linear Oscillator

1. Equilibrium Point
E.P. $x = 0$

2. Position of P.P at t $x(t)$

3. $V(t) = \frac{dx(t)}{dt}$

4) $a(t) = \frac{dV(t)}{dt}$

5) Amplitude $A = X_{max}$

6) T, $f = \frac{1}{T}$, $\omega = 2\pi f$

7) Restoring Force

$\vec{F} = -k\vec{x}$

8) $F = ma$

9) $-kx = m\frac{d^2x}{dt^2}$

$$\theta(t) = \theta_0 \cos(\omega t + \varphi)$$

$$\omega = \sqrt{\frac{\kappa}{I}}$$

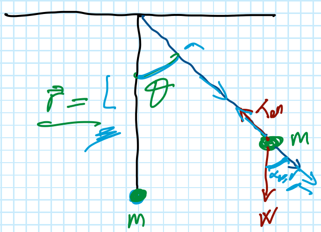
$$x(t) = A\cos(\omega t + \varphi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

10) $E_{TOT} = \frac{1}{2}kA^2$

10) $E_\theta = \frac{1}{2}\kappa\theta_0^2$

⇒ Simple Pendolum



$\theta = \alpha_{w,r}$

$$\boxed{\tau = -\kappa\theta}$$

$-\kappa\theta(t) = I\alpha = I\frac{d^2\theta(t)}{dt^2}$

$\tau = I\alpha$

$\theta(t) = \theta_0\cos(\omega t + \varphi)$

$\omega = \sqrt{\frac{\kappa}{I}}$

$I = r^2 m = L^2 m$

$\tau = \vec r \times \vec F^{net} = \vec r \times (\vec T_{en} + \vec W)$

$\vec F = \vec T_{en} + \vec W$

$|\tau| = \vec r \times \vec T_{en} + \vec r \times \vec W$

$|\tau| = \cancel{|r||T_{en}|\sin(\alpha_{T,r})} + |r||W|\sin(\alpha_{w,r})$ (180, 0)

$|\tau| = |L mg \sin\theta|$

$\theta \ll$

$$\boxed{\tau = -Lmg\sin\theta} \approx \boxed{-Lmg\theta}$$

$$\boxed{\kappa = Lmg}$$

$\theta \sim 0$

$\sin\theta \approx \theta$

⇒ $\tau = I\alpha$ —— $\frac{d^2\theta}{dt^2}$

$$\boxed{-Lmg\underbrace{\sin\theta}_{\theta} = Lm\frac{d^2\theta}{dt^2}}$$

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{Lmg}{L^2 m}} = \sqrt{\frac{g}{L}}$$

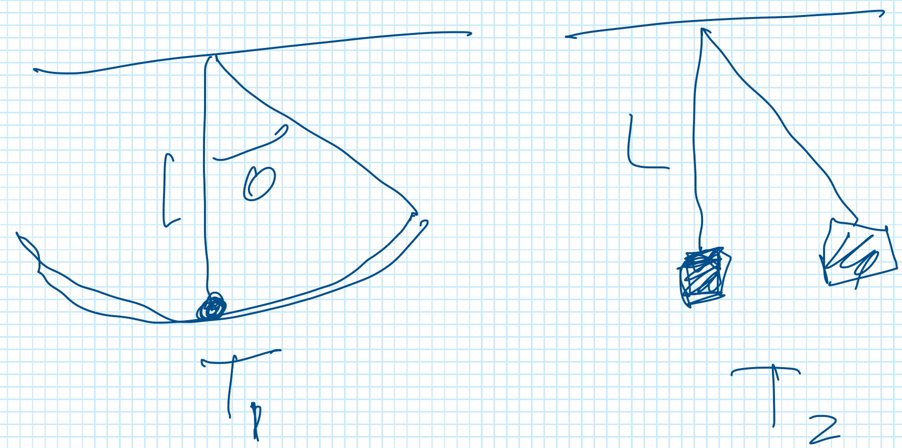
$$-g\theta = L\frac{d^2\theta}{dt^2}$$

$$\omega = \sqrt{\frac{g}{L}} = 2\pi f = \frac{2\pi}{T}$$

$$g\theta + L\frac{d^2\theta}{dt^2} = 0$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

$$\boxed{\theta(t) = \theta_0\cos(\omega t + \varphi)}$$



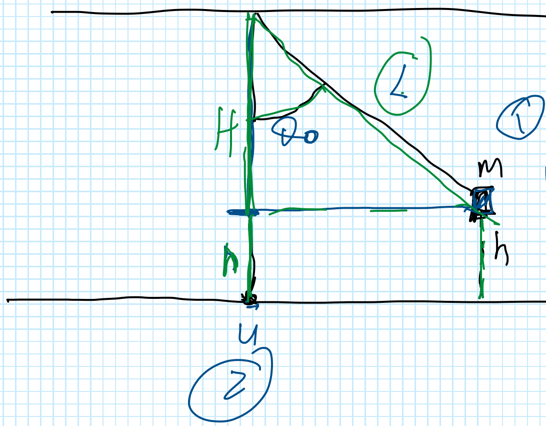
$$\omega = \sqrt{\frac{g}{L}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2\frac{L}{g} \Rightarrow \boxed{g = \frac{4\pi^2 L}{T^2}}$$

$$\boxed{E_T = \frac{1}{2}\kappa\theta_0^2 = \frac{1}{2}Lmg\theta_0^2}$$

$\Longrightarrow$ Proplem



$E_{TOT} = K + U_1$

$U = mgh$

$h = L - H$

$H + h = L$

$h = L - H$

$\boxed{H = L\cos\theta_0}$

$h = L - H = L - L\cos\theta_0$

$h = L(1-\cos\theta_0) = L(1 - \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}) = 2L\sin^2\frac{\theta}{2}$

$1 - \cos\theta_0 = 2\sin^2\frac{\theta_0}{2}$

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$\boxed{h} = 2L\sin^2\frac{\theta_0}{2} = 2L\left(\frac{\theta_0}{2}\right)^2 = 2L\frac{\theta_0^2}{4} = \boxed{\frac{L\theta_0^2}{2}}$

$\boxed{\sin\theta_0 \approx \theta_0}$

$\boxed{U_1 = mgL\frac{\theta_0^2}{2}}$ (1)

(1) $E_1 = U = mgL\frac{\theta_0^2}{2}$

$\parallel$

(2) $E_2 = K = \frac{1}{2}mV_{max}^2$

$\cancel{\frac{1}{2}}\cancel{m}V_{max}^2 = \cancel{m}gL\frac{\theta_0^2}{\cancel{2}}$

$$V_{max} = \sqrt{g\, L\, \theta_0^2} = \sqrt{g\, L} \cdot \theta_0$$