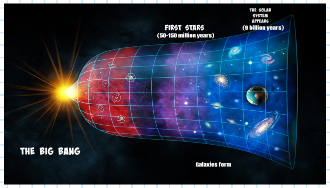
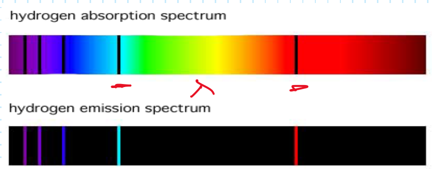
Lecture 20

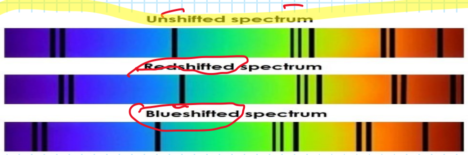
Tuesday, June 13, 2023 9:48 AM





! !!

QM

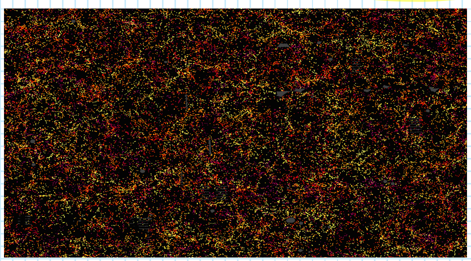


larger

$$\boxed{V(t) = H\, r(r)}$$

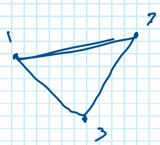
larger

Hubble const



uniform

Isotropic



$$\frac{d}{dt} r_{12}(t) = a(t)\, r_{12}(t_0)$$

$$\frac{d}{dt} r_{13}(t) = a(t)\, r_{13}(t_0)$$

$$\frac{d}{dt} r_{23}(t) = a(t)\, r_{23}(t_0)$$

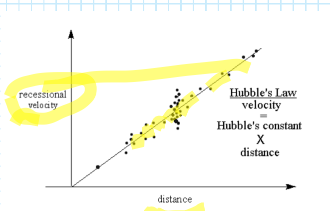
$$V_{12}(t) = \frac{\dot a}{a}\, r_{12}(t_0) \cdot a(t)$$

$$V_{13}(t) = \frac{\dot a}{a}\, r_{13}(t_0)\, a(t)$$

$$V_{23}(t) = \frac{\dot a}{a}\, r_{23}(t_0)\, a(t)$$

$$\dot a = \frac{da}{dt}$$

$$H = \frac{\dot a}{a}$$

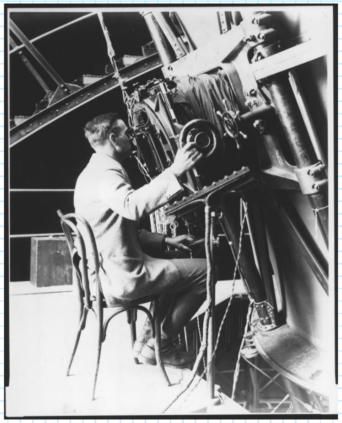


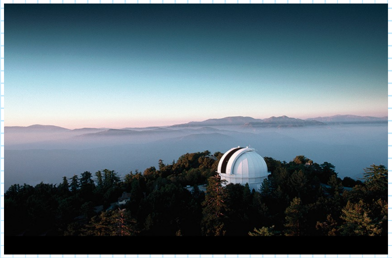
$$V_{12}(t) = H \cdot r_{12}(t)$$

$$V_{13}(t) = H\, r_{13}(t)$$

$$V_{23}(t) = H\, r_{23}(t)$$

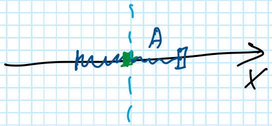
$$V(t) = H\, r(t)$$





Periodic Motion

- Spring



- Equation of Motion

$$x(t) = A\cos(\omega t+\varphi)$$

$$V(t) = \frac{dx(t)}{dt} = -\omega A\sin(\omega t+\varphi)$$

$V_{max} = \omega A$

$$a(t) = \frac{dV(t)}{dt} = -\omega^2 A\cos(\omega t+\varphi)$$

angular frequency

frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$T = \frac{1}{f}$ $\quad f = \frac{1}{T}$

φ – phase

A – Amplitude

$$\omega = \sqrt{\frac{k}{m}}$$

$$\boxed{E = K + U = \frac{1}{2}mV^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2}$$

- calculating force constant for give mass and period

$m =$

$T =$

$k = ?$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega^2 = \frac{k}{m}$$

$$\boxed{k = m\omega^2 = \frac{m4\pi^2}{T^2}}$$

$\omega = \frac{2\pi}{T}$

- for given Equation of motion of mass attached to the spring.

$x(t) = a\cos(bt + c)$ a b c

- T
- A
- f
- V_{max}
- a_{max}
- E_{TOT}
- Initial Position

$-T = ?$ $\omega = b, = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{b}$

$-A = a$

$-f = \frac{b}{2\pi}$ $f = \frac{1}{T}$

$- V_{max} = \omega A = ba$

$- a_{max} = \omega^2 A,$ $E = \frac{1}{2}kA^2$

$- E_{TOT} = \frac{1}{2}kA^2$

Initial Position

$x(0) = c\,(\;)!$

Pendulum — Simple pendulum

$\theta(t) = \theta_0 \cos(\omega t + \varphi)$

$\omega = \sqrt{\frac{g}{L}}$ κ

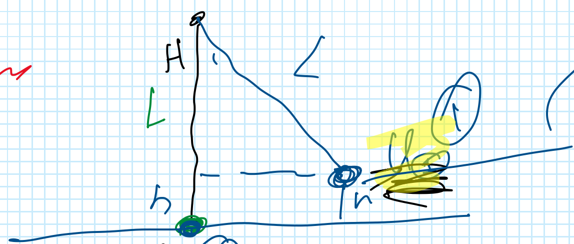
$\omega = 2\pi f$ $f = \frac{1}{T}$

Amplitude? θ_0

$\omega(t) = \frac{d\theta(t)}{dt} = -\omega\theta_0 \sin(\omega t + \varphi)$

$\alpha(t) = \frac{d\omega(t)}{dt} = -\omega^2\theta_0 \cos(\omega t + \varphi)$

$E_{TOT} = \frac{1}{2}\underbrace{\kappa}_{\kappa = Lmg}\theta_0^2 = \frac{1}{2}Lmg\theta_0^2$



$E_{TOT}^{(1)} = K_1 + U_1 = U_1$

Problem K(2)

- Given θ_0, L, m
- Calculate V_{max} Potential Energy

$U = mgh$ $h = L - H$

$H = L\cos\theta_0$

$h = L - L\cos\theta_0$

$mgh =$

$mgL(1 - \cos\theta)$

$mgL\,2\sin^2\frac{\theta}{2} =$

$\boxed{U = mgL\frac{\theta_0^2}{2}}$

V_{max}

at the Bottom $K = \frac{1}{2}mV_{max}^2$

$K_2 = U_1$

$\frac{1}{2}mV_{max}^2 = U$

Solve it for V_{max}

$\boxed{U = \frac{1}{2}mV_{max}^2}$

- $E_{TOT} = \frac{1}{2} m g \theta_0^2 = U_{(\ell)} \Big| = \frac{1}{2} m V_{max}$

$g\theta_0^2 = V_{max}$

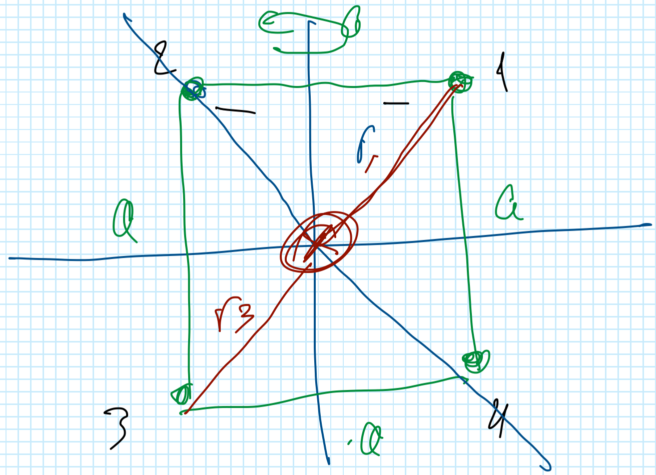
$V_{max}^2 = g\ell\theta_0^2$

$V_{max} \sqrt{gL}\ \theta_0$

- $f = ?$, $T = ?$

$m_1 =$
$m_2 =$
$m_3 =$
$m_4 =$

- Problem



$$I = \sum_{i=1}^{4} m_i r_i^2 =$$

(a) $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$

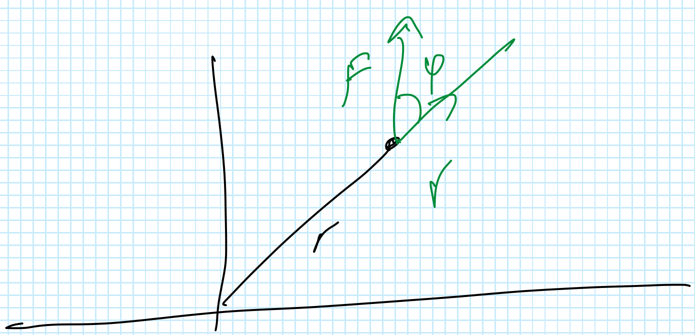
$r_1 = \frac{a}{2}$ $r_2 = \frac{a}{2}$ $r_3 = \frac{a}{2}$ $r_4 = \frac{a}{2}$

(b) $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$

$r_1 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}$ $r_2 = 0$ $r_4 = 0$

$r_3 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}$

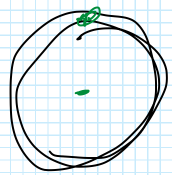
(c)



$\vec{\tau} = \vec{r} \times \vec{F}$

$|\tau| = |r|\,|F| \sin\varphi$

Problem
- grinding wheel



$t_1 = 0$

$\omega_i = 20\,\frac{rad}{s}$

$\alpha_i = 34\,\frac{rad}{s^2}$

$t_2 = 2s$

$\theta_2 = 400\,rad$ — stops

$\theta(t) = \theta_i + \omega_i t + \frac{\alpha t^2}{2}$

$\omega(t) = \omega_i + \alpha t$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

1) $\alpha_2 = ?$

$\omega_f = 0$

$0 = \omega_2^2 + 2\alpha_2\Delta\theta$

$\alpha_2 \Leftarrow$

solve it for α_2!

$\omega_2 = \omega_i + \alpha_1 t_2$

$\Delta\theta = \theta_2$

2) $\overset{0}{\omega(t_3)} = \omega_2 + \alpha_2 t_3$

solve it for t_3

$t_{total} = t_2 + t_3$

3) $\theta_1 = [\ ]$

$\frac{\theta_1 + \theta_2}{2\pi}$ — integer

$[\theta(t)] = \omega_i t + \alpha_i t^2$

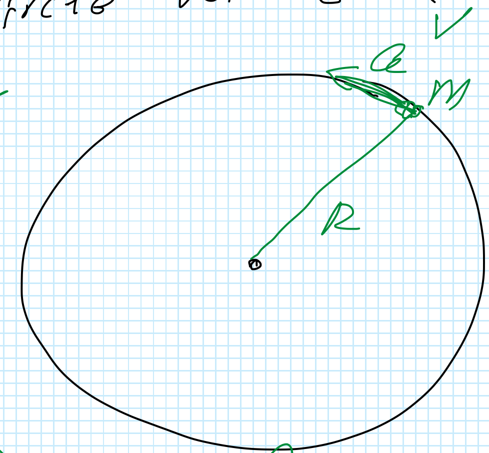
$\boxed{\theta_1(t_2)} = \omega_i t_2 + \frac{\alpha t^2}{2}$

— partical with given mass moving @ circle with given radius, a, v, t

$R\omega = v$ $\quad \omega_i = \frac{v_i}{R}$

$R\alpha = a$

$\alpha = \frac{a}{R}$



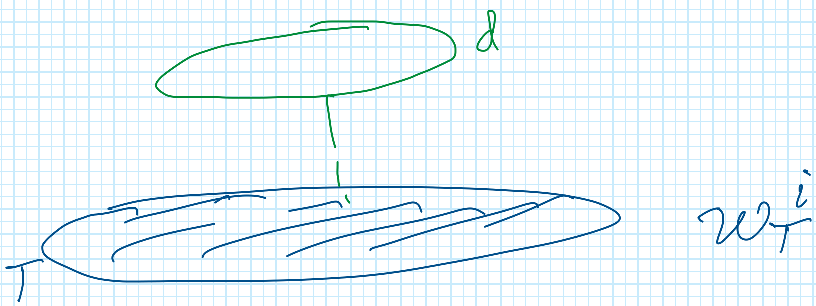
$\tau = I\alpha$

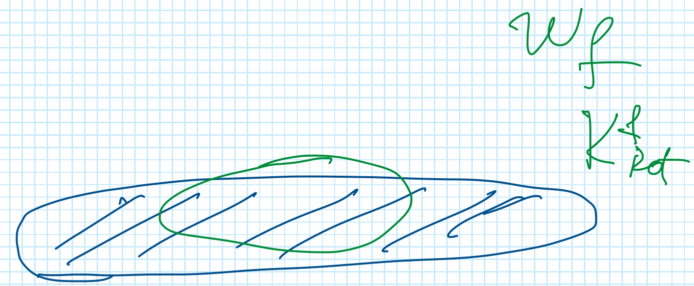
$I = mR^2$

$\alpha = \frac{a}{R}$

$\theta_f = \theta_i + \omega_i t + \frac{\alpha t^2}{2}$ $\quad\Big|\quad \frac{\theta}{2\pi} =$

—





$\Rightarrow$ Conservation of Angular Momentum

$$\boxed{L_i = L_f}$$

$L_i = L_d^i + L_T^i$ $L_f = L_{Td}$ $\boxed{I_d = \frac{1}{2} M_d R_d^2}$

$L = I\omega$

$L_i = I_d \omega_d^i + I_T \omega_T^i$

$L_f = (I_d + I_T)\,\omega_f$

$I_{disk} = \frac{1}{2} m R^2$

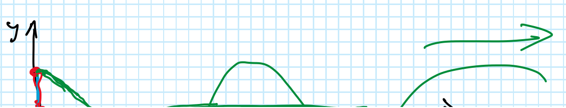
$\boxed{I_T = \frac{1}{2} m_T R_T^2}$

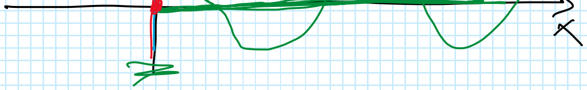
$L_f = (I_d + I_T)\,\omega_f$

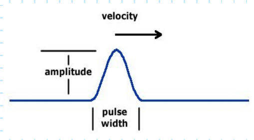
$I_T \omega_T^i = (I_d + I_T)\,\omega_f$ | Solve it for ω_f

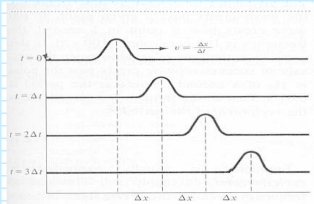
- Linear Motion 1d, 2d } Kinematics, Dynamics } Equation of Motion
- Rotational Motion } Kin, Dynamics } Eq. Motion
- Periodic Motion } Kinematics, Dynamics } Eq. Motion
- Mechanical Waves } Kinematics, Dynamics } Equation of Motion

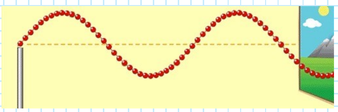
- Waves → Motion of Disturbance in the space
- Waves are transporting Energy, Power
- Create a wave create Disturbance in Time

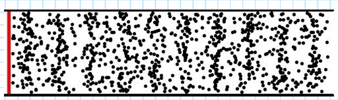












$$g = G\frac{M_p}{R_p^2}$$