Lecture 21

Wednesday, June 14, 2023 10:01 AM

Motion

- Linear Motion 1d, 2d — Kinematics, Dynamics
- Rotational Motion 2d — Kinematics, Dynamic
- Periodic Motion — Kinematics, Dynamics (Restoring Force)

→ Equation of Motion

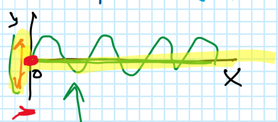
- Wave Motion

Mechanical Waves: (Sound Waves)

- Waves transport Energy and Power through the space without transporting matter itself

- Origin of waves is disturbance in time at some fixed point in the space (origin)

- Example of String

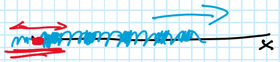


⇒ Transverse Waves

- Disturbance is perpendicular or transverse to the propagation of the wave

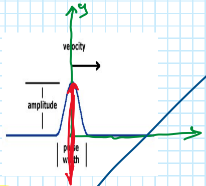
⇒ Longitudinal Waves

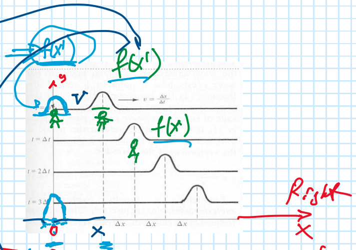
- Disturbance is in the direction of propagation of the wave



- Mathematical Issue is to relate Disturbance in time to Wave Propagation

- Simple Possible wave: Wave Pulse





⇒ Right

$x' = x - vt$

$f(x - vt) \rightarrow$ Wave Function

$\boxed{x = x' + v \cdot t}$! Right

$\boxed{x = x' - vt}$! Left

$x' = x + vt$

→ Left

$f(x+vt) \longrightarrow$ Wave Function

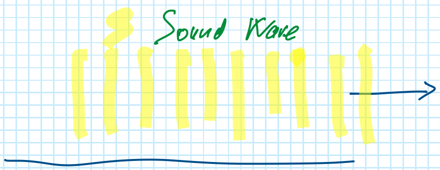
$f(x - vt)$ Right $\qquad f(x+vt)$ Left

– Summarizing

– v – speed of Propogation

$\boxed{y(x,t)} = f(x \pm vt)$ $+ \to$ propogating Left; $- \to$ propogating Right

$\longrightarrow$ Transverse Waves



$\longrightarrow$ Longitudinal wave

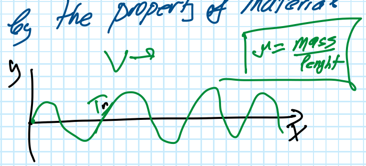
https://www.surendranath.org/GPA/Waves/TW01/TW01.html

https://www.sciencelearn.org.nz/videos/1857-demonstrating-longitudinal-and-transverse-waves

⇒ Speed of Waves

– V – characterized by the property of material

String



$V = \sqrt{\dfrac{T_n}{\mu}}$ – String

– Sound Waves

fluids $\qquad V = \sqrt{\dfrac{B}{\rho}}$ $\qquad \rho$ – equilibrium density

$\underline{B} = -\dfrac{\Delta P}{\Delta V / V}$ – Bulk Modulus

gases $\qquad V = \sqrt{\dfrac{\gamma RT}{\ldots}}$ $\qquad T$ – temperature

Universal

$R = 8.314 \frac{J}{mol \cdot K}$ – Universal Gas Const

M – molar Mass

γ – type of the gas

$\gamma = 1.4$ O_2

$\gamma = 1.67$ He

$V \sim$ Elasticity of the Medium

Harmonic Waves

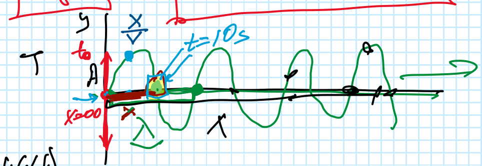
Perturbation is Harmonic

Spring $x(t) = A\cos(\omega t + \varphi)$

Pendulum $\theta(t) = \theta_0 \cos(\omega t + \varphi)$

Wave Length

$$\boxed{\lambda = V \cdot T}$$



$$y(0,t_0) = A\cos(\omega t_0 + \varphi)$$

$$\boxed{\omega = 2\pi f, \quad T = \frac{1}{f}}$$

$$t_0 = t - \frac{x}{V}$$

$$y(x,t) = A\cos\left(\omega\left(t - \frac{x}{V}\right) + \varphi\right)$$

$$y(x,t) = A\cos\left(\omega t - \frac{\omega}{V}x + \varphi\right)$$

– introduce – wave Number $\boxed{k = \frac{\omega}{V}}$

$A\cos(kx - \omega t - \varphi)$ $\boxed{-\varphi = \delta}$

$$\boxed{y(x,t) = A\cos(\omega t - kx + \varphi)}$$ Harmonic wave Right

$\boxed{\cos(\alpha) = \cos(-\alpha)}$

$$y(x,t) = A\cos(\omega t + kx + \varphi)$$ Harmonic wave Left

$A\cos(kx + \omega t + \varphi)$

$\omega = 2\pi f$

$T = \frac{1}{f}$

$$\boxed{k = \frac{\omega}{V}} = \frac{2\pi f}{V} = \frac{2\pi}{V \cdot T} = \frac{2\pi}{\lambda}$$

$\lambda = T \cdot V$

wave Number.

$- A =$

$\Rightarrow$ Wave Equation

$$\boxed{y(x,t) = A\cos(kx - \omega t + \delta)}$$

$$\frac{dy(x,t)}{dx} = -kA\sin(kx - \omega t + \delta)$$

$$\boxed{\frac{d^2y(x,t)}{dx^2} = -k^2 A\cos(kx - \omega t + \delta)} \Rightarrow$$

$$\frac{dy(x,t)}{dt^2} = \omega A\sin(kx - \omega t + \delta)$$

$$\boxed{\frac{d^2y(x,t)}{dt^2} = -\omega^2 A\cos(kx - \omega t + \delta)\frac{k^2}{k^2}} \Rightarrow$$

$$\boxed{\frac{d^2y(x,t)}{dt^2} = \frac{\omega^2}{k^2}\frac{d^2y(x,t)}{dx^2}}$$

$$\frac{\omega^2}{k^2} = \frac{\cancel{\omega^2}}{\omega^2/v^2} = v^2$$

$$\boxed{\frac{d^2y(x,t)}{dt^2} = v^2\frac{d^2y(x,t)}{dx^2}}$$ Wave Equation

$$\boxed{\frac{d^2y(x,t)}{dx^2} = \frac{1}{v^2}\frac{d^2y(x,t)}{dt^2}}$$

v = propagation

$$F = k\frac{q_1q_2}{r^2}$$

$E(x,t)$ $\qquad$ $B(x,t)$

$$F = \mu\frac{v_1v_2q_1q_2}{r^2}$$

$$\boxed{\frac{d^2E(x,t)}{dx^2} = \frac{1}{v^2}\frac{d^2E(x,t)}{dt^2}}$$

$c = c$

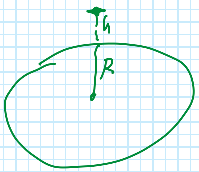
$$\boxed{v = \frac{1}{\sqrt{k\mu}}} = c$$

$y(x,t) = 10_m \cos(3x + 2t + 3)$

- Amplitude
- frequency
- wave number
- Period
- velocity of propagation
- direction of propagation
- phase

⟹ Gravity.

- Escape velocity of Satellite on orbit



$E_s = U_G + K = -\frac{G M_p m_s}{R_p + h} + \frac{1}{2} m_s v^2$

$U_s = -\frac{G M_p m_s}{R_p + h}$

$K = \frac{1}{2} m_s v^2$

$E = 0$

solve for

v

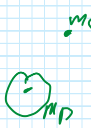
⟹ Calculate mass of the planet if you know

$a_{surface}$

$g = \frac{G M_E}{R_E^2}$

M_p

⟹ Calculate period of satellite



$F = \frac{G M_p m_s}{r^2} = m_s \cdot a_c$

$a_c = \frac{v^2}{r}$

v

$\frac{2\pi r}{v} = T$

ω

$\theta(t) = \theta_0 \cos(\omega t + \varphi)$

$\theta(0) = \theta_0 \cos\varphi$