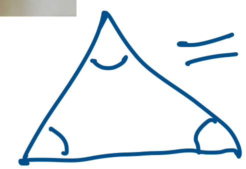
Lecture 3

2:18 PM

⇒ We live in 3d space:
In a good approximation this space is flat

323-283 BC



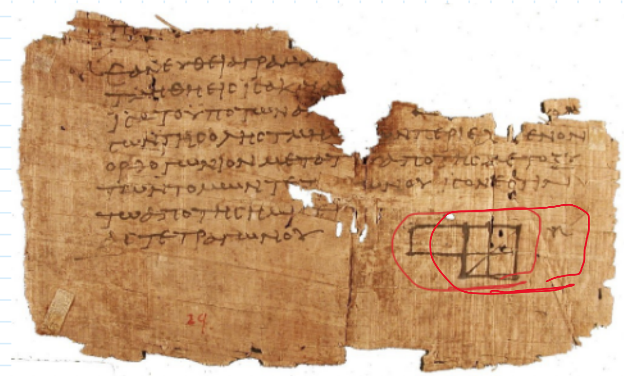
$= 180°$

Euclid's Axioms

1. A straight line may be drawn between any two points.
2. Any terminated straight line may be extended indefinitely.
3. A circle may be drawn with any given point as center and any given radius.
4. All right angles are equal.

5'. For any given point not on a given line, there is exactly one line through the point that does not meet the given line.

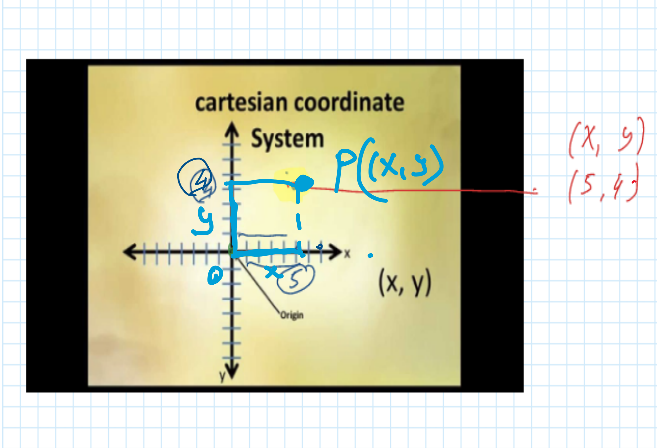
Stanze di Raffaello, in the Apostolic Palace 1509-1511



100 AD

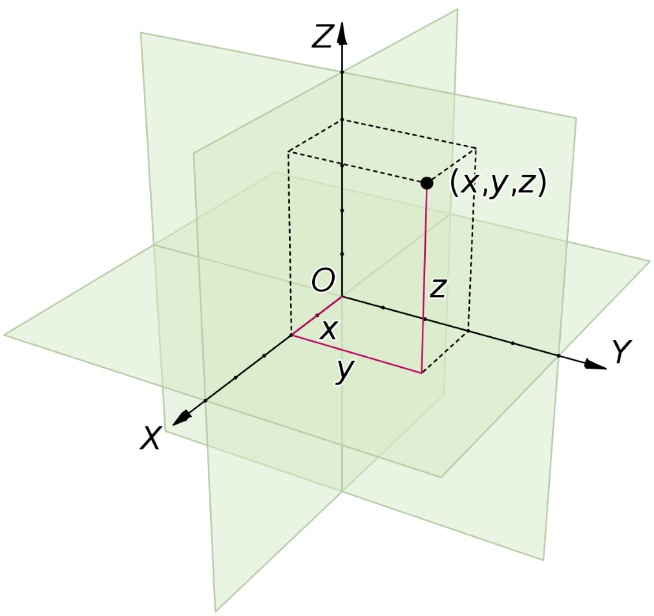
Cartesian Coordinate Systems

Cartesian Coordinate Systems



(x, y)
(5, 4)

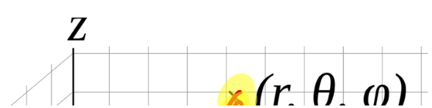
Rene Descartes (French) or **Renatus** Cartesius
1595 - 1650

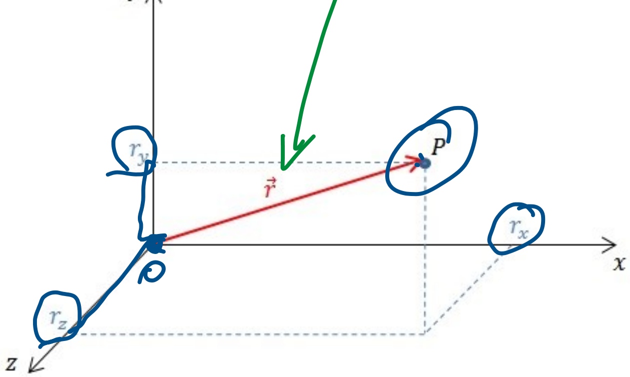


Right handed

Vectors

Components
Cartesian

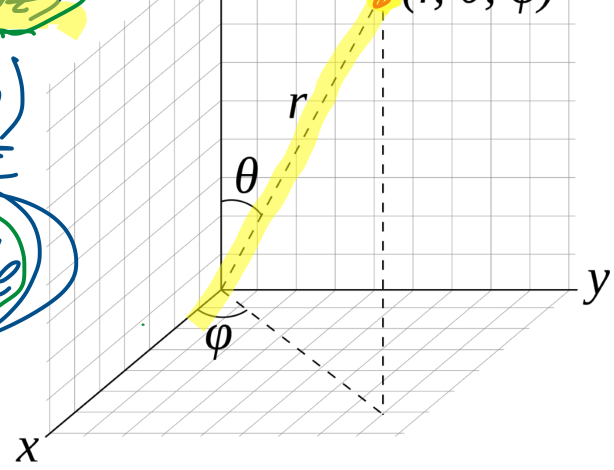




$\vec{r} = (r_x, r_y, r_z)$

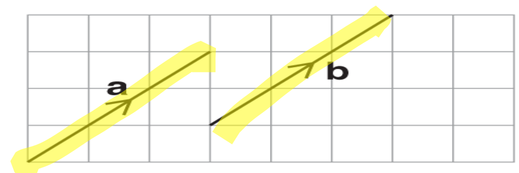
$\vec{r} = (r, \theta, \varphi)$

polar coordinate

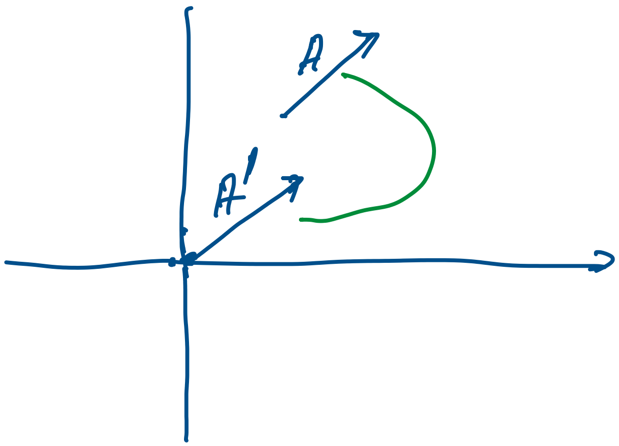


$\vec{A}$ $|\vec{A}|$

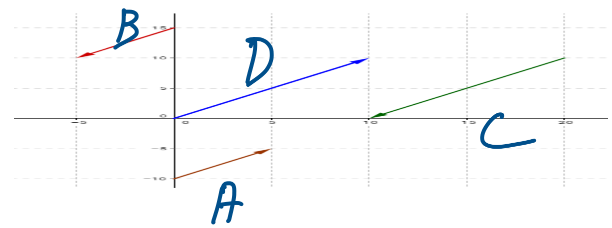
1. Vector has magnitude and directions:



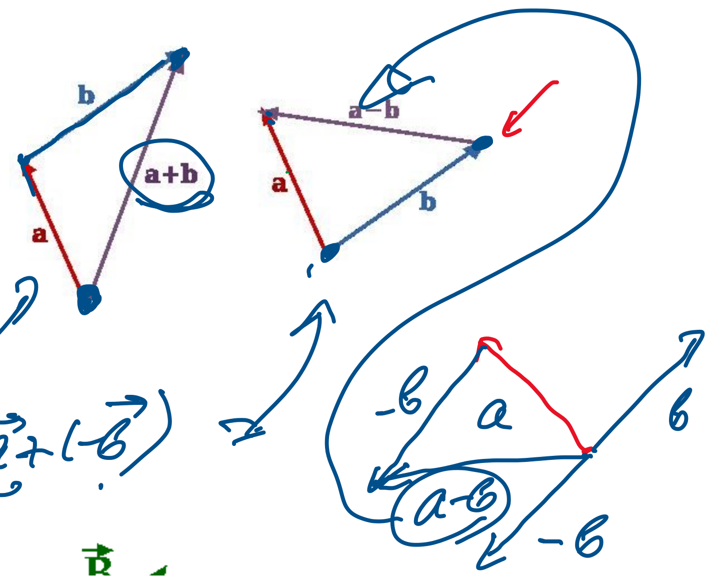
- Vectors with same magnitudes and directions are identical



$\vec{A} = -\vec{B}$



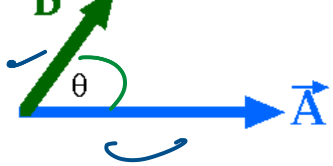
- Summing and Subtracting Vectors



$\vec{a} + \vec{b} =$

$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

- Scalar Products of Vectors



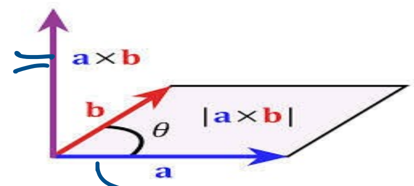
$$\underbrace{\vec{A}\cdot\vec{B}}_{\text{Number}} = \underbrace{|\vec{A}||\vec{B}|\cos\theta}_{\text{Number}}$$

- Cross Product

$$\vec{C} = \vec{a}\times\vec{b}$$

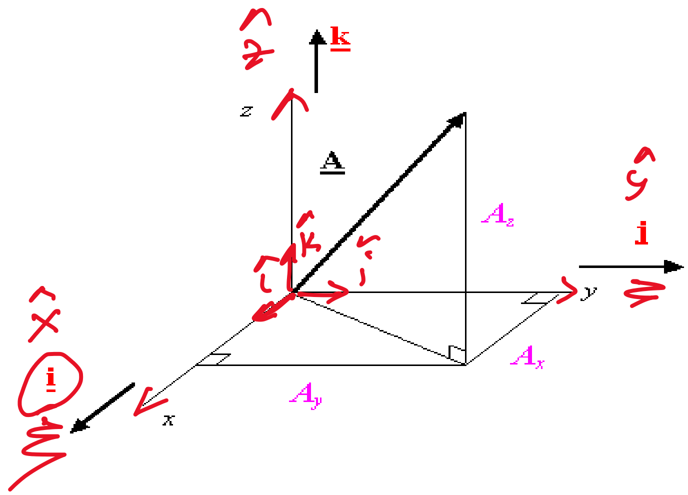
Vector

$\vec{C} =$



$\vec{C}$

$$\boxed{|\vec{C}| = |\vec{a}||\vec{b}|\sin\theta}$$



— Unit vector $|\vec{i}| = 1$ $\qquad \hat{x} \equiv \hat{i} \qquad \hat{y} \equiv \hat{j} \qquad \hat{z} \equiv \hat{k}$

$$\hat{i}\cdot\hat{j} = \underbrace{|i||j|}_{1\cdot 1}\cos\overset{90^\circ}{\theta_{ij}} = 0$$

$$\hat{i}\cdot\hat{k} = 0 \qquad \hat{k}\,\hat{j} = 0$$

$$\rightarrow \vec{A} = (|\vec{A}|, \theta, \varphi) = (A_x \; A_y \; A_z) \equiv (A_x\cdot\hat{i} + A_y\hat{j} + A_z\hat{k})$$

$\hat{x} \qquad \hat{y} \qquad \hat{z}$

$\rightarrow \vec{A} = \vec{B}$

$|A| = |B|$

$\theta_A = \theta_B$

$\varphi_A = \varphi_B$

$A_x = B_x$

$A_y = B_y$

$A_z = B_z$

$\rightarrow \vec{A}\cdot\vec{B} = |A||B|\cos\theta_{AB} = A_x\cdot B_x + A_y B_y + A_z B_z$

$\rightarrow \vec{A}\times\vec{B} = \vec{C} = (C_x\hat{i} + C_y\hat{j} + C_z\hat{k})$

$\hat{x}$ $\hat{y}$ $\hat{z}$

$C_x = A_y B_z - A_z B_y$

$x \quad y \quad z \quad z \quad y$

$y \quad z \quad x \quad - x \quad z$

$C_y = A_z B_x - A_x B_z$

$z \quad x \quad y$

$C_z = A_x B_y - A_y B_x$

$\rightarrow \vec{A} = (|A|, \theta, \varphi)$ $\vec{A} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$

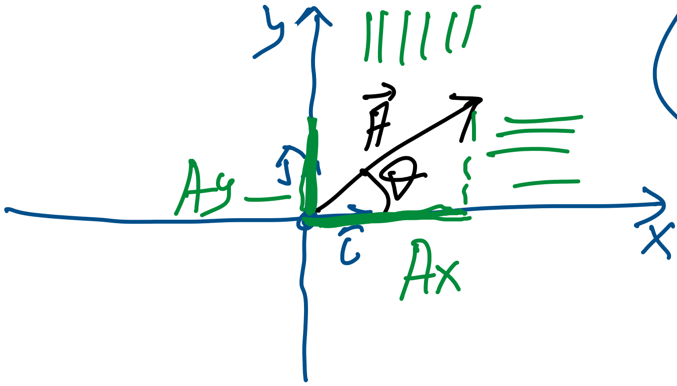
$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$A_z = |A|\cos\theta$

$A_x = |A| \sin\theta \cos\varphi$

$A_y = |A| \sin\theta \sin\varphi$

2d- case

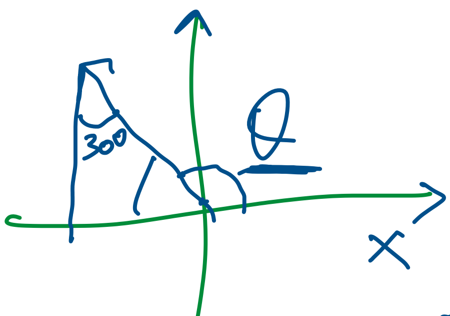


$\vec{A} = (|\vec{A}|, \theta) = (A_x \hat{i} + A_y \hat{j})$

$$\boxed{|A| = \sqrt{A_x^2 + A_y^2}}$$

$A_x = |A| \cos\theta$

$A_y = |A| \sin\theta$



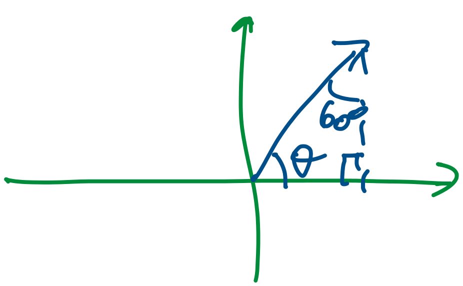
$\underline{\theta}$

$$\frac{A_y}{A_x} = \frac{\cancel{|A|}\sin\theta}{\cancel{|A|}\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\boxed{\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)}$$

$\theta = 120$

$A_x = |A| \cos\theta = |A| \cos 120$



$\theta = 30°$

$30 + \theta + 90 = 180$

$\theta = 60^\circ$

5d

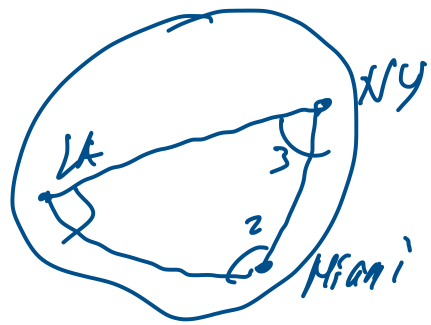
$$\vec{A} = (A_1\hat{1} + A_2\hat{2} + A_3\hat{3} + A_4\hat{4} + A_5\hat{5})$$

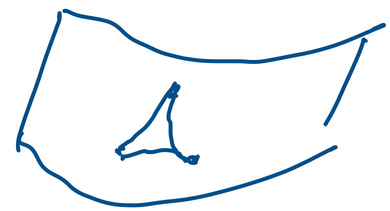
$$|A| = \sqrt{A_1^2 + A_2^2 + A_3^2 + A_4^2 + A_5^2}$$

— Euclidean Space

$\theta_1 + \theta_2 + \theta_3$

— Curved



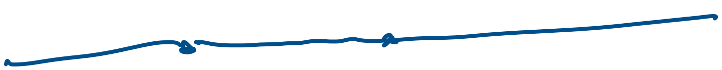


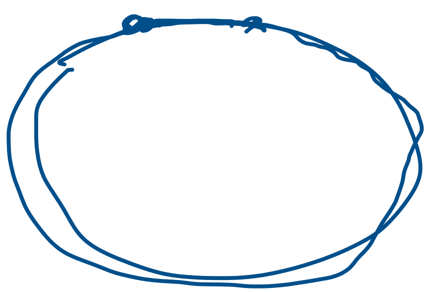
+ curvature

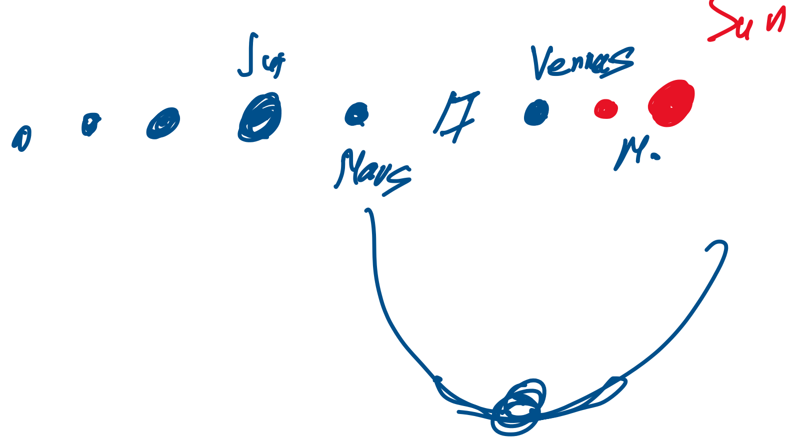
Rieaman

– curvature

Lobachevsics







Linear

2d - motion