Lecture 4

Thursday, May 11, 2023 11:07 AM

2d Motion

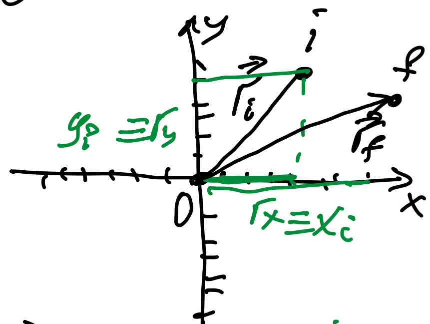
$\vec{A} = (A_x \hat{i} + A_y \hat{j})$

$\vec{B} = (B_x \hat{i} + B_y \hat{j})$

$\boxed{\vec{A} - \vec{B} = ((A_x - B_x)\hat{i} + (A_y - B_y)\hat{j})}$

① Motion of P. O.

1. Reference Frame
   Reference Point



2- Initial Position $\vec{r}_i = (x_i \hat{i} + y_i \hat{j})$

2' Initial time t_i

3, Final Position $\vec{r}_f = (x_f \hat{i} + y_f \hat{j})$

3' Final Time t_f

4. Displacement $\boxed{\Delta \vec{r} = \vec{r}_f - \vec{r}_i = (\Delta x \hat{i} + \Delta y \hat{j})}$

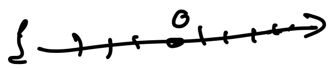
4' Time Interval $\Delta t = t_f - t_i$

5 Average velocity $\boxed{\vec{V}_{AV}} = \dfrac{\Delta \vec{r}}{\Delta t} = \left(\dfrac{\Delta x}{\Delta t}\hat{i} + \dfrac{\Delta y}{\Delta t}\hat{j}\right)$, with $V_{AV}^{x} = \dfrac{\Delta x}{\Delta t}$, $V_{AV}^{y} = \dfrac{\Delta y}{\Delta t}$

$\boxed{(V_{AV}^{x} \hat{i} + V_{AV}^{y} \hat{j})}$

1d

0 P. O



2. Initial Position x_i

2' Initial time t_i

3. Final Position

3' Final time

4. Displacement $\Delta x = x_f - x_i$

4' Time Interval $\Delta t = t_f - t_i$

5) Average Velocity $V_{AV} = \dfrac{\Delta x}{\Delta t}$

6) Average Acceleration

$a_{AV} = \dfrac{\Delta V}{\cdot}$

$V_{ins} = \frac{dr}{dt} = \left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right)$ Δt

$(V_x \hat{i} + V_y \hat{j})$

6) Average Acceleration

$$\vec{a} = \frac{\Delta \vec{V}}{\Delta t} = \left(\overbrace{\frac{\Delta V_x}{\Delta t}}^{a_x}\hat{i} + \overbrace{\frac{\Delta V_y}{\Delta t}}^{a_y}\hat{j}\right)$$

$$\boxed{\vec{a} = (a_x \hat{i} + a_y \hat{j})}$$

Equation of Motion

2d

Galelee
horizontal and
Vertical Motions
are Independent

1) 2 Babys Version

$\vec{a} = 0$

x

$$\boxed{\begin{gathered} V_i^x = V_f^x = V_x \\ X_f = X_i + V_x \cdot \Delta t \end{gathered}}$$

y

$$\boxed{\begin{gathered} V_i^y = V_f^y = V_y \\ y_f = y_i + V_y \cdot t \end{gathered}}$$

1d

1) Baby Version

$a = 0$

$V_i = V_f = V$

$X_f = X_i + V \cdot \Delta t$

2) Junior Version

$a = 0$

$V_f = V_i + a\Delta t$

$$X_f = X_i + V_i \Delta t + \frac{a\Delta t^2}{2}$$

2) 2 Junior Versions

$\boxed{X}$

$$V_f^x = V_i^x + a_x \cdot \Delta t$$

$$X_f = X_i + V_x \Delta t + \frac{a_x \Delta t^2}{2}$$

$\boxed{y}$

$$V_f^y = V_i^y + a_y \Delta t$$

$$Y_f = y_i + V_i^y \Delta t + \frac{a_y \Delta t^2}{2}$$

3) Adult version

$$V_f = \int_{t_i}^{t_f} a\, dt$$

$$X_f = \int_{t_i}^{t_f} V\, dt$$

3) 2. Adult versions

$\boxed{X}$

$$V_f^x = \int_{t_i}^{t_f} a_x\, dt$$

$$X_f = \int_{t_i}^{t_f} V_x\, dt$$

$\boxed{y}$

$$V_f^y = \int_{t_1}^{t_2} a_y\, dt$$

$$Y_f = \int_{t_1}^{t_2} V_y\, dt$$

$$2\Delta X \cdot a = V_f^2 - V_i^2$$

$$2\Delta X\, a_x = {V_f^x}^2 - {V_i^x}^2$$

$$2\Delta y\, a_y = {V_f^y}^2 - {V_i^y}^2$$

$\longrightarrow$ Running Time

$X_f \rightarrow X(t)$

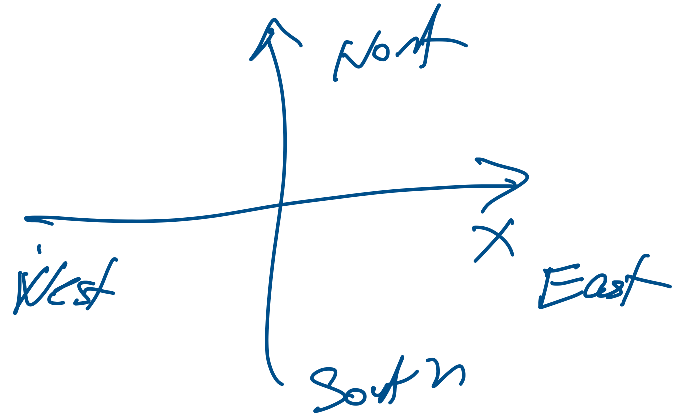
$\Delta t \longrightarrow t$

$V_f = V(t)$

x $\qquad$ y

$x^f \to x(t)$ $\qquad$ $y^f = y(t)$

$V_x^f = V(t)$ $\qquad$ $V_y^f = V(t)$



Example:

1d

$t_i = 0$ $\qquad$ $V_i = 60 \frac{mi}{h}$ $\qquad$ $a = \frac{V_f - V_i}{\Delta t} = 0$

$t_f = 2h$ $\qquad$ $V_f = 60 \frac{mi}{h}$

2d

$t_i = 0$ $\qquad$ East $\qquad$ $60 \frac{mi}{h}$ $\qquad$ $a = ?$

$\Delta t = 2h$

$t_f = 2h$ $\qquad$ North $\qquad$ $60 \frac{mi}{h}$

$\vec{\ }$ = (...) = $\left(60 \frac{mi}{h} \hat{i} + 0 \hat{j} \right)$

$$\vec{V_i} = (V_x^i \hat{i} + V_y^i \hat{j}) = (\ \ldots$$

$$\vec{V_f} = (V_x^f \hat{i} + V_y^f \hat{j}) = (0\, \hat{i} + 60 \tfrac{mi}{h} \hat{j})$$

$$\Delta\vec{V} = \vec{V_f} - \vec{V_i} = \left((V_x^f - V_x^i)\hat{i} + (V_y^f - V_y^i)\hat{j} \right) =$$

$$\left(-60 \tfrac{mi}{h} \hat{i} + 60 \tfrac{mi}{h} \hat{j} \right)$$

$$\vec{a} = \frac{\Delta\vec{V}}{\Delta t} = \left(-\frac{60 \frac{mi}{h}}{2h} \hat{i} + \frac{60 \frac{mi}{h}}{2h} \hat{j} \right) =$$

$$= \Big(\underbrace{-30 \tfrac{mi}{h^2}}_{a_x} \hat{i} + \underbrace{30 \tfrac{mi}{h^2}}_{a_y} \hat{j} \Big)$$

$$|a| = \sqrt{a_x^2 + a_y^2} = \sqrt{\underbrace{(-30)^2}_{900} + \underbrace{(30)^2}_{900}} \frac{mi}{h^2} = \sqrt{1800} \frac{mi}{h^2}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{\ldots}\right.$$

$\theta = \tan^{-1}\left(\frac{\quad}{a_x}\right)$

$$\vec{A} = (A_x\hat{i} + A_y\hat{j})$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$