Lecture 5

Monday, May 15, 2023 9:54 AM

- Physical Quantities
  Magnitude
- Units, SI
  - Fundamental Unit
  - Derived
- Dimension
- Mechanics
  → Motion — Kinematics / Dynamics

- 1d. Linear Motion
- Point object
- Reference Frame
- Position in Space, magnitude, m ± x_i
- Position in Time t_i, s, +
- Displacement $\Delta x = x_f - x_i$, m, ±
- Time interval $\Delta t = t_f - t_i$, s +
- Distance s
- Average velocity $V_{AV} = \frac{\Delta x}{\Delta t}$, $\frac{m}{s}$ ±
- Speed $= \frac{s}{\Delta t}$, $\frac{m}{s}$ +
- Instantaneous Velocity $V = \frac{dx}{dt} = \frac{\Delta x}{\Delta t}\Big|_{\Delta t \to 0}$
- Average Acceleration $a_{AV} = \frac{\Delta V}{\Delta t}$, $\frac{m}{s^2}$, ±
- Instantaneous Acceleration $a = \frac{dV}{dt}$

- Concept of truth: Prediction
- Equation of Motion
- Baby version $a = 0$

$$x_f = x_i + V \cdot \Delta t$$
$$V_f = V_i = V$$

- Junior version $a = \text{const}$

$$V_f = V_i + a\,\Delta t$$
$$x_f = x_i + V_i \Delta t + \frac{a\,\Delta t^2}{2}$$
$$2a\Delta x = V_f^2 - V_i^2$$

- Adult version $a \neq \text{const}$

$$V_f = \int_{t_i}^{t_f} a(t)\,dt$$
$$x_f = \int_{t_i}^{t_f} V(t)\,dt$$

⇒ Running time

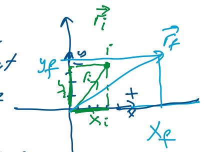
$t_i = 0$, $t_f = t$ | $x_f = x(t)$, $V_f = V(t)$

- Universal Constant
  c, h
- Natural Units
  c = 1
  h = 1
  Energy
- Quantum Photon

E. Radiation $E = h\nu$

2d Motion

0. Point object
1. Reference Frame



2. Position in 2d space
   position vector $\vec{r} = (x\,\hat{x} + y\,\hat{y})$, $\hat{x} = \hat{i}$, $\hat{y} = \hat{j}$

2. Time t

3. Displacement vector

$$\vec{r}_f = (x_f\,\hat{x} + y_f\,\hat{y})$$
$$\vec{r}_i = (x_i\,\hat{x} + y_i\,\hat{y})$$
$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = (\underbrace{(x_f - x_i)}_{\Delta x}\,\hat{x} + \underbrace{(y_f - y_i)}_{\Delta y}\,\hat{y})$$
$$\Delta\vec{r} = (\Delta x\,\hat{x} + \Delta y\,\hat{y})$$

3' time interval $\Delta t = t_f - t_i$

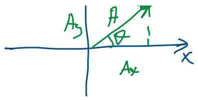
4. Average Velocity vector

$$\vec{V}_{AV} = \frac{\Delta\vec{r}}{\Delta t} = \left(\underbrace{\frac{\Delta x}{\Delta t}}_{V_{AV}^x}\hat{x} + \underbrace{\frac{\Delta y}{\Delta t}}_{V_{AV}^y}\hat{y}\right)$$
$$\vec{V}_{AV} = (V_{AV}^x\,\hat{x} + V_{AV}^y\,\hat{y})$$
$$\vec{V} = \left(\frac{dx}{dt}\,\hat{x} + \frac{dy}{dt}\,\hat{y}\right)$$

5.
$$\vec{a}_{AV} = \frac{\Delta\vec{V}}{\Delta t} = (a_{AV}^x\,\hat{x} + a_{AV}^y\,\hat{y})$$
$$a = \frac{d\vec{V}}{dt} = \left(\frac{dV_x}{dt}\,\hat{x} + \frac{dV_y}{dt}\,\hat{y}\right)$$

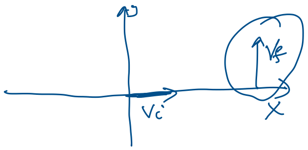
- 2d Motion
- Vectors:
- Cartesian Coordinate System

$$\vec{A} = (|\vec{A}|, \theta) = (A_x\,\hat{x} + A_y\,\hat{y}) = (A_x\,\hat{i} + A_y\,\hat{j})$$



$A_x = |A|\cos\theta$
$A_y = |A|\sin\theta$

$|A| = \sqrt{A_x^2 + A_y^2}$
$\theta = \tan^{-1}\frac{A_y}{A_x}$



$|V_i| = 60\frac{mi}{h}$ East
$t_i = 0$
$t_f = 2h$
$|V_f| = 60\frac{mi}{h}$ North

i $V_i^x = V_i$
$V_i^y = 0$
$\vec{V_i} = (V_i\hat{x} + 0\hat{y})$

$\vec{V_f} = (0\hat{x} + V_f\hat{y})$

$\vec{a} = \frac{\vec{V_f} - \vec{V_i}}{\Delta t} = \frac{((V_f^x - V_i^x)\hat{x} + (V_f^y - V_i^y)\hat{y})}{\Delta t}$

$\vec{A} = (A_x\hat{x} + A_y\hat{y})$
$\vec{B} = (B_x\hat{x} + B_y\hat{y})$
$\vec{A} - \vec{B} = ((A_x - B_x)\hat{x} + (A_y - B_y)\hat{y})$

$= \frac{-60\frac{mi}{h}}{2h}\hat{x} + \frac{60\frac{mi}{h}}{2h}\hat{y} =$

$\vec{a} = (\underbrace{-30\frac{mi}{h^2}}_{a_x})\hat{x} + \underbrace{30\frac{mi}{h^2}}_{a_y}\hat{y}$

$|a| = \sqrt{(-30)^2 + (30)^2}\ \frac{mi}{h^2} = \sqrt{1800}\ \frac{mi}{h^2}$
$a_x^2 + a_y^2$

$\theta_a = \tan^{-1}\frac{a_y}{a_x}$

⟹ Equation of Motion
One 2d motion = two 1d motions

1) Baby Version $\vec{a} = 0$ $a_x = 0$, $a_y = 0$

(x)
$V_x^f = V_x^i = V_x$
$\boxed{x_f = x_i + V_x \cdot \Delta t}$

(y)
$V_y^f = V_y^i = V_y$
$y_f = y_i + V_y \cdot \Delta t$

2) Junior Version $\vec{a} = const$ $a_x = const$ $a_y = const$

(x)
$V_x^f = V_x^i + a_x\Delta t$
$x^f = x^i + V_x^i \cdot \Delta t + \frac{a_x \Delta t^2}{2}$
$2a_x\Delta x = V_x^{f2} - V_x^{i2}$

(y)
$V_y^f = V_y^i + a_y\Delta t$
$y^f = y^i + V_y^i\Delta t + \frac{a_y\Delta t^2}{2}$
$2a_y\Delta y = V_y^{f2} - V_y^{i2}$

3) Adult Version

$V_x^f = \int_{t_i} a_x dt$ $\qquad$ $V_y^f = \int_{t_i} a_y dt$

$x^f = \int_{t_i}^{t_f} V_x dt$ $\qquad$ $y^f = \int_{t_i}^{t_f} V_y dt$

⇒ Running Time $\Delta t = t$

x

$V_x(t) = V_i^x + a_x t$

$X(t) = X_i + V_i^x t + \frac{a_x t^2}{2}$

y

$V_y(t) = V_i^y + a_y t$

$Y(t) = y_i + V_i^y t + \frac{a_y t^2}{2}$

$\boxed{t \text{ - is common}}$

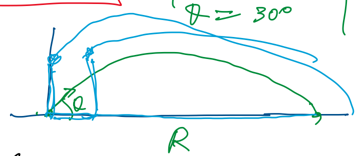
$|V_i| = 20\frac{m}{s}$

$\theta = 30°$

$V_y^i = |V_i| \sin\theta$

$V_y^i = V_i \sin\theta$

$V_x = |V_i| \cos\theta$



⇒ Projectile Motion

1) Total Time that the Ball is in the air

2) Horizontal Distance it travelled (Range)

x

$a_x = 0$

$V_x^i = V_x^f = V_x$

$X(t) = X_i + V_x \cdot t$

0

y

$a_y = -g$ $\qquad$ $g = 9.8\frac{m}{s^2}$

$V_y(t) = V_y^i + a_y t$

$y(t) = y_i + V_y^i \cdot t + \frac{a_y t^2}{2}$

0

$y(t_{total}) = 0 = V_y^i t_{total} + \frac{a_y t_{total}^2}{2} = 0$

$t_{total}\left(V_y^i + \frac{a_y t_{total}}{2}\right) = 0$

$a \cdot b = 0$

$a = 0, \quad b = 0$

$t_{total} = 0$

$V_y^i + \frac{a_y t_{total}}{2} = 0$

$a_y t_{total} = -2V_y^i$

$\boxed{t_{total}} = \frac{-2V_y^i}{a_y} = \boxed{\frac{2V_i \sin\theta}{g}}$

$a_y = -g$

$R = X(t_{total}) = V_x \frac{2V_i \sin\theta}{g}$

$V_x = V_i \cos\theta$

$R = V_i \cos\theta \, 2 V_i \sin\theta$

$$\sin 2\theta$$

$$V_{yi} = V_i \sin\theta$$

$$R = \frac{V_i^2\, 2\sin\theta\cos\theta}{g}$$

$$\boxed{R = \frac{V_i^2 \sin 2\theta}{g}}$$

90

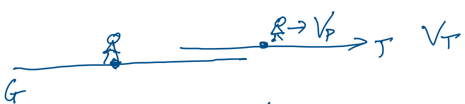
$\sin 2\theta = 1$

$\theta = 45^\circ$

$\sin 90 = 1$

Relative Motion

1d



$$V_G = V_P + V_T$$

Detroit $\quad X_G = V_G \cdot t = V_P \cdot t + V_T \cdot t$

50 mi/h

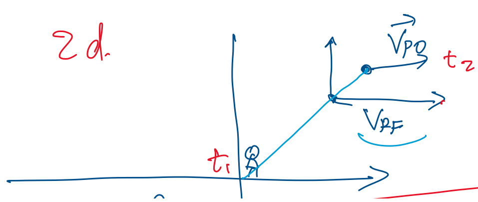
Miami

$$V_G = \boxed{V_{R/Air}} + V_{Air}$$

$t_2 < t_1$

$t_1 \neq t_2$

2d.



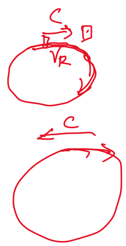
G

$$\vec{V}_G = \vec{V}_{PO/RF} + \vec{V}_{RF}$$

$$\vec{r}_G = \vec{r}_{PO/RF} + \vec{V}_{RF} \cdot t$$

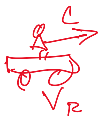
Galilean Relativity

$C = 3\times10^8 \frac{m}{s}$

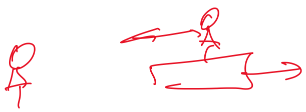


$V_1 = C + V_R$

$V_2 = C - V_R$

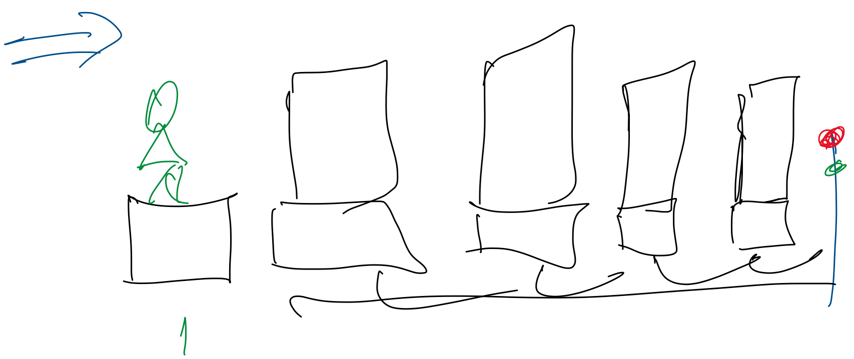


$V_1 = C + V_R$



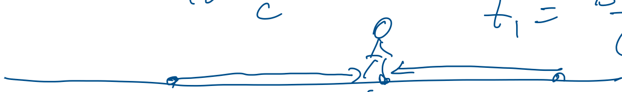
$V_1 = V_2$

$V_1 = C - V_R$



$t_2 = \frac{\Delta x_2}{c}$

$t_1 = \frac{\Delta x_1}{c}$

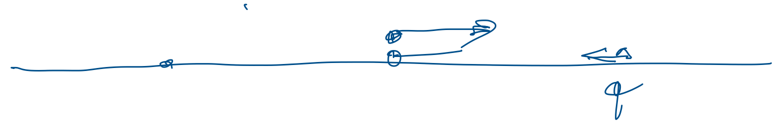


Δx Δx_1

$\Delta x_1 <$

$t_1 = \frac{\Delta x_1}{c}$

Δx_2



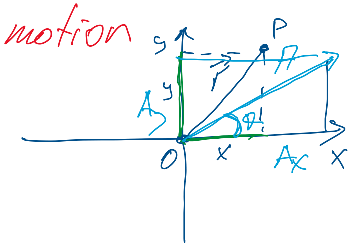
$t_2 = \frac{\Delta x_2}{c}$

$t_2 > t_1$

Summary:

2d- motion

– Vectors:



$\vec{r} =$

$\vec{A} = (A_x \hat{i} + A_y \hat{j})$

$\hat{x}$ $\hat{y}$

$\vec{A}$; $|A|$ θ

$A_x = |A| \cos\theta$

$A_y = |A| \sin\theta$

$\vec{A} = (A_x \hat{i} + A_y \hat{j})$

$\vec{B} = (B_x \hat{i} + B_y \hat{j})$

$\vec{A} + \vec{B} = \vec{C} = (\underbrace{(A_x + B_x)}_{C_x}\hat{i} + \underbrace{(A_y + B_y)}_{C_y}\hat{j})$

$|A| = \sqrt{A_x^2 + A_y^2}$

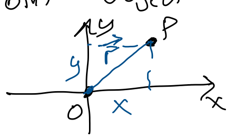
$\theta = \tan^{-1}\frac{A_y}{A_x}$

$\vec{A} - \vec{B} = \vec{D} = (\underbrace{(A_x - B_x)}_{D_x}\hat{i} + \underbrace{(A_y - B_y)}_{D_y}\hat{j})$

2d- Kinematics

1. Point Object

2.



3. Position of Point object

$\vec{r} = (x\hat{i} + y\hat{j})$, m

3'. Time t s

4. Displacement Vector

3. Displacement vector

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \underbrace{(x_f - x_i)}_{\Delta x}\hat{i} + \underbrace{(y_f - y_i)}_{\Delta y}\hat{j} = (\Delta x\,\hat{i} + \Delta y\,\hat{j})$$

4. Time Interval $\Delta t = t_f - t_i$

5. Average Velocity $$\vec{V}_{AV} = \frac{\Delta\vec{r}}{\Delta t} = \Big(\overbrace{\frac{\Delta x}{\Delta t}}^{V_{AV}^{x}}\hat{i} + \overbrace{\frac{\Delta y}{\Delta t}}^{V_{AV}^{y}}\hat{j}\Big) = (V_{AV}^{x}\hat{i} + V_{AV}^{y}\hat{j})$$

$$\vec{V} = \Big(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\Big) = (V_x\hat{i} + V_y\hat{j})$$

6. Average Acceleration $$\vec{a}_{AV} = \frac{\Delta\vec{V}}{\Delta t} = \Big(\overbrace{\frac{\Delta V_x}{\Delta t}}^{a_x}\hat{i} + \overbrace{\frac{\Delta V_y}{\Delta t}}^{a_y}\hat{j}\Big) = (a_{AV}^{x}\hat{i} + a_{AV}^{y}\hat{j})$$

$$\vec{a} = \Big(\frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j}\Big) = (a_x\hat{i} + a_y\hat{j}) \qquad \vec{a} = \frac{\Delta\vec{V}}{\Delta t}\Big|_{\Delta t \to 0} = \frac{d\vec{V}}{dt}$$

Equation of Motion

- time is the same in all directions
- Motions in x and y directions are independent

One 2d motion = Two 1d motions in x and y directions

x – direction

1) Baby Version $a_x = 0$

$$V_f^x = V_i^x = V_x$$

$$x_f = x_i + V_x \cdot \Delta t$$

y – direction

1) Baby version $a_y = 0$

$$V_f^y = V_i^y = V_y$$

$$y_f = y_i + V_y \cdot \Delta t$$

$V_x = \text{const}$

$x(t) = x_i + V_x \cdot t$

2) Junior Version $a_x = \text{const}$

$$V_f^x = V_i^x + a_x \cdot \Delta t$$

$$x_f = x_i + V_i^x \Delta t + \frac{a_x \Delta t^2}{2}$$

$$V_x(t) = V_x + a_x t$$

$$x(t) = x_i + V_i^x t + \frac{a_x t^2}{2}$$

3) Adult Version

$$V_f^x = \int_{t_i}^{t_f} a_x \, dt = V_x(t)$$

$$x_f = \int_{t_i}^{t_f} V_f^x \, dt = x(t)$$

$V_y = \text{const}$

$y(t) = y_i + V_y \cdot t$

~~2) Junior Version $a_y = \text{const}$~~

$\boxed{a_y = -g}$

$$V_f^y = V_i^y + a_y \Delta t$$

$$y_f = y_i + V_i^y \Delta t + \frac{a_y \Delta t^2}{2}$$

$$V_y(t) = V_i + a_y t$$

$$y(t) = y_i + V_i^y t + \frac{a_y t^2}{2}$$

3) Adult Version

$$V_f^y = \int_{t_i}^{t_f} a_y \, dt = V_y(t)$$

$$y_f = \int_{t_i}^{t_f} V_f^y \, dt = y(t)$$

Running time $t_i = 0$, $t_f = t$ | $\Delta t = t$

$x_f = x(t)$

$y_f = y(t)$

$$\theta = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$

$$\Rightarrow \quad \vec{r}(t) = \left(x(t)\,\hat{i} + y(t)\,\hat{j} \right)$$

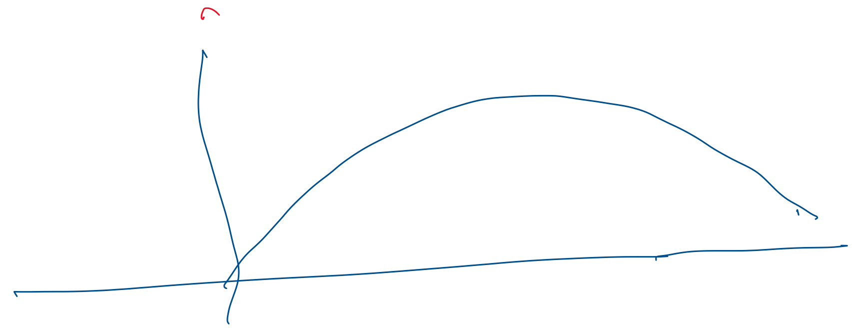
$$|r(t)| = \sqrt{x(t)^2 + y(t)^2}$$

$$\vec{A} = \left(A_x \hat{i} + A_y \hat{j} \right)$$

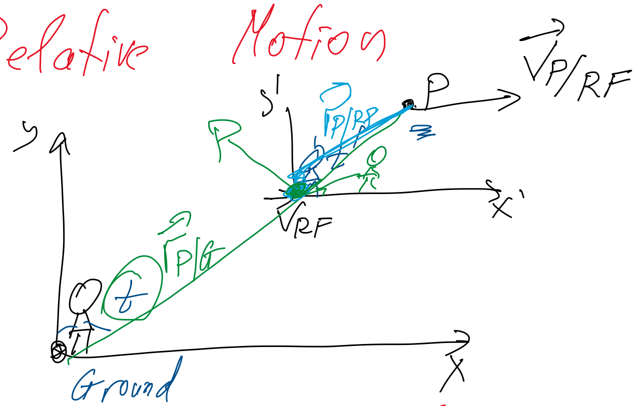
$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$|V(t)| = \sqrt{V_x^2(t) + V_y^2(t)} \qquad \theta_v = ?$$

Projectile Motion



Relative Motion



Galilean Relativity

$t = t'$

$\vec{r}_{P/G} = \vec{r}_{P/RF} + \vec{V}_{RF} \cdot t$

Example

Speci

Theo

Rel

$$V_{P/G} = V_{P/RF} + V_{RF}$$

$$V_{P/RF} = v_0 \qquad \vec{V}_{RF} = \vec{V}_0 \qquad V_{P/G} = 2v_0$$

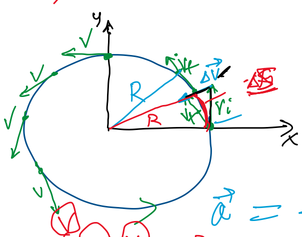
$$\left.\begin{aligned}\vec{V}_{P/RF} &= \vec{V}_0 \\ \vec{V}_{RV} &= -\vec{V}_0\end{aligned}\right\} \quad \vec{V}_{P/G} = 0$$

⟹ Motion In a Circle

speed

$|V| = \text{const} = V$

$\vec{a} = \dfrac{\Delta \vec{V}}{\Delta t}$



$$\vec{a} = \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t}$$

$$a = \frac{\Delta V}{\Delta t} = \frac{\Delta V}{V}\cdot\frac{V}{\Delta t} = \frac{\Delta S\, V}{R\,\Delta t} = \frac{V^2}{R}$$

$$\frac{\Delta V}{V} = \frac{\Delta S}{R}$$

$\boxed{\text{Centripetal}}$ Acceleration Points to the center.

$$\boxed{a = \frac{V^2}{R}}$$