Lecture 8

Thursday, May 18, 2023 10:13 AM

- Newton's Three Laws

Law 1. In Inertial Reference Frames

if $\vec{F}_{net} = 0$ then $\vec{V}_{object} = \frac{const}{0}$ (all the forces)

Law 2 If $\vec{F}_{net} \neq 0$

$\vec{F}_{net} = m\,\vec{a}_{object}$

$[F_{net}] = kg\frac{m}{s^2} = N$

Law 3 $\vec{F}_{reaction} = -\vec{F}_{action}$

$\vec{F}_{reaction}$ $\vec{F}_{action}$

— Contact Forces: — Spring Force $\vec{F} = -k\Delta x$ ↳ Force Contact

↳ Hooks law

— String-force → $\vec{T}$, tension

— Normal-force $\vec{N}$

— Friction Forces $f_s = \mu_s N$, $f_k = \mu_k N$

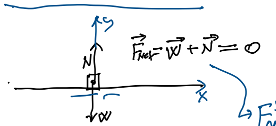
⇒ Force → Inverse Problem

Newton 2nd law $\vec{F} = m\,\vec{a}_{object}$ | $\vec{a}_{object}\, m = \vec{F}$

1) Weight m $\vec{a} = -g$ $|a| = g$ → $-g\,m = \vec{F}$

$\vec{F} = -m\vec{g}$

$\boxed{|F| = mg = W}$ weight

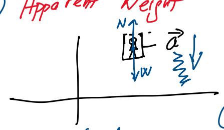


$\vec{F}_{net} = \vec{W} + \vec{N} = 0$

↳ $F^y_{net} = -mg + N = 0$

$\boxed{N = mg}$

2) Apparent Weight



$\vec{F}_{net} = \vec{W} + \vec{N} = m\vec{a}$

ⓐ $F^y_{net} = -mg + N = -m\,a_y$ ↓

ⓑ $F^y_{net} = -mg + N = ma$ a_y ↑

a) a points down

$N = mg - ma_y = m(g - a)$

Free fall $a = g$ $\boxed{N = 0}$

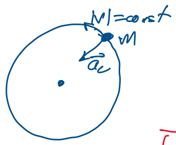
b) a point up

$N = mg + ma = m(g + a)$

↑ $a = g$ $N = 2mg$

$a = 9g$ $N = 10mg$ $\boxed{10g}$

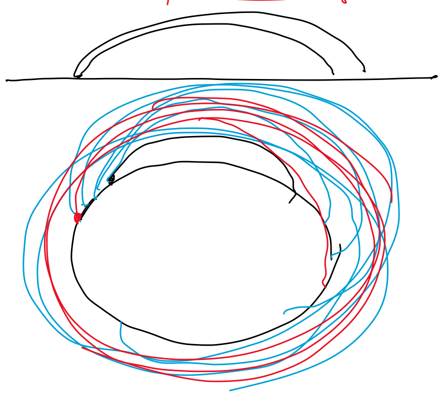
⇒ Centripetal Force

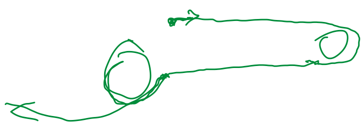


$\vec{a}_c = \frac{v^2}{R} \longrightarrow$ center

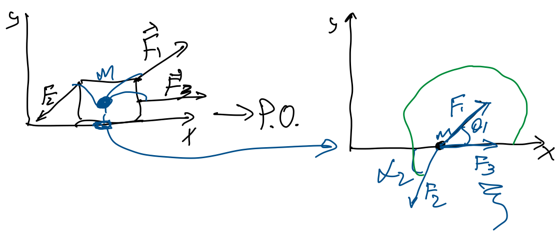
$\vec{F}_c \longrightarrow$ center

$$\boxed{\vec{F}_c = m\vec{a}_c}$$





⇒ Free Body Diagrams.



θ - defined with respect to positive x

① $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

ⓐ $\vec{F}_{Net} = 0$ – Equilibrium Problem

ⓑ $\vec{F}_{Net} = m\vec{a}$ – Non Equilibrium

→ Project in x and y directions

$$F_{Net}^{x} = F_1^{x} + F_2^{x} + F_3^{x} = ||$$

$$|| = |F_1|\cos\theta_1 + |F_2|\cos(\alpha_2 + 180) + |F_3|$$

$$\underbrace{\cos(\alpha_2+180)}_{-\cos\alpha_2}$$

$$F_{Net}^{x} = |F_1|\cos\theta_1 - |F_2|\cos\alpha_2 + F_3$$

(y)

$$F_{Net}^{y} = F_1^{y} + F_2^{y} + F_3^{y} =$$

$$= |F_1|\sin\theta_1 + |F_2|\underbrace{\sin(\alpha_2+180)}_{-\sin\alpha_2}$$

$$F_{Net}^{y} = |F_1|\sin\theta_1 - |F_2|\sin\alpha_2$$

Equilibrium

$$F_{Net}^{x} = |F_1|\cos\theta_1 - |F_2|\cos\alpha_2 + F_3 = 0$$

$$F_{Net}^{y} = |F_1|\sin\theta_1 - |F_2|\sin\alpha_2 = 0$$

Non Equilibrium

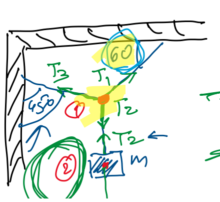
$$F_{Net}^{x} = |F_1|\cos\theta_1 - |F_2|\cos\alpha_2 + F_3 = m a_x$$

$$F_{Net}^{y} = |F_1|\sin\theta_1 - |F_2|\sin\alpha_2 = m a_y$$

Solve for unknown quantity

⇒ Connected Several Objects

⇒ FBD for each object



$T_3 = 30N$

$\alpha_1 = 60°$

$\alpha_3 = 45°$

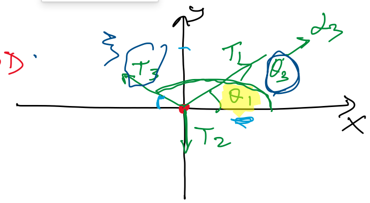
$T_2 = ?$ →

$T_1 = ?$

$\boxed{m = ?}$ →

$\downarrow W$ two – object

① FBD:



$\boxed{\theta_1 = \alpha_1}$

$\theta_3 = 180 - \beta_3$

$\beta_3 = 90 - \alpha_3$

$\boxed{\theta_3 = 90 + \alpha_3}$

!

$$\vec{F}_{net} = \vec{T}_1 + \vec{T}_2 + \vec{T}_3 = 0$$

x

$$F_{net}^x = T_1^x + T_2^x + T_3^x = 0$$

$$F_{net}^x = T_1 \cos\theta_1 + 0 + T_3 \cos\theta_3 = 0$$

$$T_1 \cos\theta_1 + T_3 \cos\theta_3 = 0$$

$$T_1 \cos\theta_1 = -T_3 \cos\theta_3$$

$$\boxed{T_1} = -T_3 \frac{\cos\theta_3}{\cos\theta_1} = -T_3 \frac{\cos(90+\alpha_3)}{\cos\alpha_1} = \boxed{\frac{T_3 \sin\alpha_3}{\cos\alpha_1}}$$

X-projection

$T_1 = [\ ]$

y

$$F_{net}^y = T_1^y + T_2^y + T_3^y$$

$$\boxed{F_{net}^y} = T_1 \sin\theta_1 - T_2 + T_3 \sin\theta_3 = 0$$

$$\cos(90+\alpha_3) = \cos 90 \cdot \cos\alpha_3 - \sin 90 \sin\alpha_3 = -\sin\alpha_3$$

$$\cos(\beta_1+\beta_2) = \cos\beta_1 \cos\beta_2 - \sin\beta_1 \sin\beta_2$$

$\beta_1 = 90 \quad \beta_2 = \alpha_3$

$-\sin\alpha_3$

T_3^X

$$T_3 \cos\theta_3 = T_3 \cos(90+\alpha_3) = -T_3 \sin\alpha_3$$

y

$$F_{net}^y = T_1 \sin\theta_1 - T_2 + T_3 \sin\theta_3$$

$$T_2 = T_1 \sin\theta_1 + T_3 \sin\theta_3$$

$$\theta_1 = \alpha_1$$

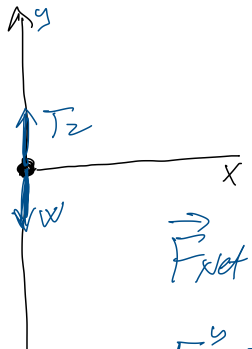
$\theta_3 = 90 + \alpha_3$

$$\boxed{T_2 = T_1 \sin\alpha_1 + T_3 \underbrace{\sin(90+\alpha_3)}_{\cos\alpha_3}}$$

$\sin(\beta_1+\beta_2) = \sin\beta_1\cos\beta_2 + \cos\beta_1\sin\beta_2$

$\sin 90+\alpha_3 = \cos\alpha_3$

2

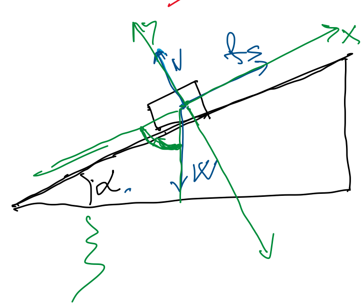


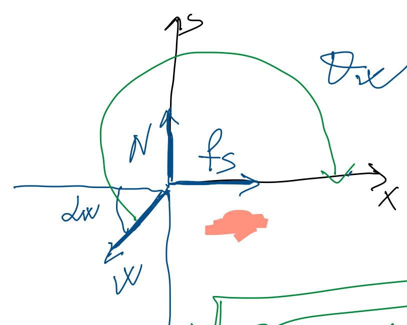
$\vec{F}_{net} = \vec{T}_2 + \vec{W}$

$F^y_{net} = T_2 - mg = 0$

$mg = T_2 \Rightarrow \boxed{m = \dfrac{T_2}{g}}$

$\mu_s = ?$





$\alpha_w = 90 - \alpha$

$$\boxed{\theta_w = \alpha_w + 180°}$$

$\alpha_w = 90 - \alpha$

$$\vec{F}_{net} = \vec{N} + \vec{f}_s + \vec{W} = 0$$

x

$$F^x_{net} = 0 + f_s + mg\cos\theta_W$$

x

$$f_s + mg\cos\theta_W = 0$$

$$\cos(\alpha_W + 180^\circ) = -\cos\alpha_B$$

$$\mu_s \cdot N + mg\cos\theta_W = 0$$

$$\mu_s N \cancel{+} mg\cos\alpha_W = 0$$

$$\mu_s N = mg\cos\alpha_W$$

μ_s

$$\mu_s = \frac{mg\cos\alpha_W}{N} = \frac{mg\cos\alpha_W}{N} = \frac{mg\sin\alpha}{N} = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$$

y

$$F^y_{net} = N + 0 + mg\sin\theta_W = 0$$

$(\alpha_W + 180^\circ)$

$$N + mg\sin\theta_W = 0$$

$$N = -mg\sin\theta_W = mg\sin\alpha_W = mg\cos$$

$$\alpha_W = 90 - \alpha$$

$\sin\alpha$

$\cos(90 - \alpha)$

$$\boxed{\mu_s = \tan\alpha}$$