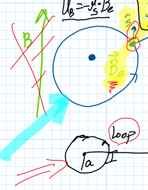
L12

Monday, March 1, 2021 4:59 PM

⇒ Inherent Spin Orbit Interaction $\frac{\hbar}{2m} e L_z \cdot B$ (m)

$U_B = -\vec{\mu}_s \cdot \vec{B}_e$



$\boxed{\vec{\mu}_s = \frac{\hbar e}{m} \vec{S}}$

$H = \frac{p^2}{2m} - \frac{ze^2}{r} - \vec{\mu}_s \cdot \vec{B}_e =$

$\boxed{\vec{\mu}_e = \frac{\hbar e}{2m} \vec{L}}$

$\boxed{B_e = \frac{\mu_e}{\pi r^3}}$

$\boxed{\vec{\mu} = -\frac{e\hbar}{2m}\vec{L}}$

$\boxed{\frac{-e^2 \vec{S}\cdot\vec{L}}{(2m)^2 r^3}}$

$\mu_z = I \cdot A$

$B_z = \frac{\mu_0 I a^2 \pi}{\pi 2 (z^2+a^2)^{3/2}} \Big|_{z \to 0} = \boxed{\frac{\mu_z}{2\pi r^3}}$

$B_z = \frac{\mu_0 I a^2 \pi}{\pi 2 (z^2+a^2)^{3/2}}$

$\vec{B} = \frac{\vec{\mu}}{\pi r^3}$

$H = \underbrace{\frac{p^2}{2m} - \frac{ze^2}{r}}_{H_0} - \frac{\hbar^2 e^2}{(2m)^2 \pi r^3} \vec{S}\cdot\vec{L}$ Spin-orbital Interaction

$\boxed{[H_0, L_i] = 0 \quad [H_0, L^2] = 0}$

$\boxed{[H L_i] \neq 0}$

$[H L_i] = \underbrace{[H_0 L_i]}_{0} - \frac{e^2}{(2m)^2} \boxed{[\vec{S}\cdot\vec{L}, L_i]}$

$\cancel{[\hat{L}, L_i] = 0}$

$J_i = S_z \quad [S_z] = \Sigma S_z^i = \frac{1}{2}$

$\frac{1}{2}$

$\boxed{[\vec{S}\cdot\vec{L}, L_i]} = \sum_j [S_j L_j, L_i] =$

$\sum_j (S_j L_j L_i - L_i S_j L_j) =$ (commute)

$= \sum_j (S_j L_j L_i - S_j L_i L_j) =$

$= \sum_j S_j (L_j L_i - L_i L_j) =$

$[L_i L_j] = i \sum \varepsilon_{ijk} L_k$

$= \boxed{-\sum_{jk} S_j \varepsilon_{ijk} L_k} \neq 0$

HW $\boxed{[\vec{S}\cdot\vec{L}, S_i] \neq 0}$

$\boxed{\hat{J}_i = \hat{L}_i + \hat{S}_i}$!

Total Angular Momentum

$[H, J_i] = 0$

$[\vec{S}\cdot\vec{L}, J_i] = 0$

$\boxed{J_i = L_i + S_i \qquad J^2 = J_x^2 + J_y^2 + J_z^2}$

HW

$[H, L_z] \neq 0 \qquad [H, S_z] \neq 0$

$[H, J_z] = 0$ — m_j

$[H, J^2] = 0$ — j

$[H, L^2] = 0$ → ℓ

$[H, S^2] = 0$ → S

not know m_ℓ, m_s

$[L_i L_j] = i \sum_k \varepsilon_{ijk} L_k$

$[L_k L_j] = i \sum_m \varepsilon_{kjm} L_m$

$L_k L_j - L_j L_k = +i \sum_m \varepsilon_{kjm} L_m$

$\boxed{[l\cdot s, L_i] \neq 0}$

$[L\cdot S, L^2] = 0 \Longrightarrow \boxed{\sum_j [L_j S_j, L^2]} = \sum_k \sum_j [L_j S_j, \hat{L}_k \cdot \hat{L}_k] =$

$\boxed{[L\cdot S, \hat{S}^2] = 0}$

$= \sum_k \sum_j (L_j S_j L_k L_k - L_k L_k L_j S_j) = \sum_k \sum_j (L_j L_k L_k S_j - L_k L_k L_j S_j) =$

$= \sum_k \sum_j (L_j L_k L_k S_j - L_k (L_j L_k + i \sum_m \varepsilon_{kjm} L_m) \cdot S_j$

$= \sum_{kj} (L_j L_k L_k S_j - L_k L_j L_k S_j - L_k i \sum_m \varepsilon_{kjm} L_m S_j$

$= \sum_{kj} (L_j L_k L_k S_j - L_k L_j L_k S_j - i \sum_{kjm} \varepsilon_{kjm} L_k L_m S_j$

$= \sum_{kj} (L_j L_k L_k S_j - L_k L_j L_k S_j)$

$= \sum_{kj} [L_j L_k] \cdot L_k S_j = i \sum_{kjm} \varepsilon_{jkm} L_m L_k S_j$

$[L_j L_k] = i \sum_m \varepsilon_{jkm} L_m$

0

Total Angular Momentum

$\boxed{\hat{J}_i = \hat{L}_i + \hat{S}_i}$

$[H J_i] = 0, \qquad [H J^2] = 0$

$[H, L^2] = 0, \quad [H, S^2] = 0$

$[J^2, L^2] = 0 \quad [J^2, S^2] = 0$

$[J_i, L^2] = 0 \quad [J_i, S^2] = 0.$

$\ell \qquad S$

$J_z |\phi_{j m_j}\rangle \quad m_j |\phi_{j, m_j}\rangle$

$J^2 |\phi_{j, m_j}\rangle = j(j+1) |\phi_{j, m_j}\rangle$

⟹ $\boxed{|\psi_{\ell, m_\ell}\rangle = Y_\ell^m(\theta, \varphi)}$

⟹ $\boxed{|\psi_{s, m_s}\rangle = |S, m_s\rangle}$ — spin quantum state = $\begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}0\\1\end{pmatrix}$

$|\phi_{j,m}\rangle$ — related to $|\psi_{\ell m}\rangle$ and $|s, m_s\rangle$