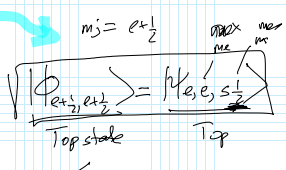


General case of l and $s = \frac{1}{2}$

$l=1, s=\frac{1}{2} \rightarrow \begin{cases} j = \frac{3}{2} \\ j = \frac{1}{2} \end{cases}$
 $l, s = \frac{1}{2} \rightarrow \begin{cases} j = l + \frac{1}{2} \\ j = l - \frac{1}{2} \end{cases} \quad |\psi\rangle = \sum x_n |l, m\rangle$

$|\phi_{l, m_j}\rangle = \sum_{m_1, m_2} C_{m_1, m_2} |\psi_{l, m_1, s, m_2}\rangle$
 $|\phi_{l, m_j}\rangle = C_{m_1 = \frac{m_j}{2}} |\psi_{l, m_1, s, \frac{m_j}{2}}\rangle + C_{m_2 = \frac{m_j}{2}} |\psi_{l, m_2, s, \frac{m_j}{2}}\rangle$
 $m_j = l + \frac{1}{2}$
 $C_{m_1 = \frac{m_j}{2}} = \alpha_m$
 $C_{m_2 = \frac{m_j}{2}} = \beta_m$



$|\phi_{l, m_j}\rangle = \alpha_m |\psi_{l, m_1 = \frac{m_j}{2}, s, \frac{m_j}{2}}\rangle + \beta_m |\psi_{l, m_2 = \frac{m_j}{2}, s, \frac{m_j}{2}}\rangle$
 $\alpha_m = \frac{\sqrt{(j+m)(j-m+1)}}{\sqrt{(l+m)(l-m)}} |\phi_{l, m-1, m-1}\rangle + \beta_m |\psi_{l, m-1, m-1, s, \frac{m-1}{2}}\rangle + \beta_m |\psi_{l, m-1, m-1, s, \frac{m-1}{2}}\rangle$
 $\alpha_m = \frac{\sqrt{(j+m)(j-m+1)}}{\sqrt{(l+m)(l-m)}} |\phi_{l, m-1, m-1}\rangle + \beta_m |\psi_{l, m-1, m-1, s, \frac{m-1}{2}}\rangle$
 $\alpha_m = \frac{\sqrt{(j+m)(j-m+1)}}{\sqrt{(l+m)(l-m)}} |\phi_{l, m-1, m-1}\rangle + \beta_m |\psi_{l, m-1, m-1, s, \frac{m-1}{2}}\rangle$

$\alpha_m L_- |\psi_{l, m-1, m-1, s, \frac{m-1}{2}}\rangle + \beta_m L_- |\psi_{l, m-1, m-1, s, \frac{m-1}{2}}\rangle + \alpha_m S_- |\psi_{l, m-1, m-1, s, \frac{m-1}{2}}\rangle + \beta_m S_- |\psi_{l, m-1, m-1, s, \frac{m-1}{2}}\rangle = 0$

$= \alpha_m \frac{\sqrt{(l+m)(l-m+1)}}{\sqrt{(l+m)(l-m)}} |\psi_{l, m-2, s, \frac{m-2}{2}}\rangle + \beta_m \frac{\sqrt{(l+m)(l-m+1)}}{\sqrt{(l+m)(l-m)}} |\psi_{l, m-2, s, \frac{m-2}{2}}\rangle + \alpha_m \frac{\sqrt{(l+m)(l-m+1)}}{\sqrt{(l+m)(l-m)}} |\psi_{l, m-2, s, \frac{m-2}{2}}\rangle + \beta_m \frac{\sqrt{(l+m)(l-m+1)}}{\sqrt{(l+m)(l-m)}} |\psi_{l, m-2, s, \frac{m-2}{2}}\rangle$

$= \alpha_m \sqrt{(l+m)(l-m+1)} |\psi_{l, m-2, s, \frac{m-2}{2}}\rangle + \beta_m \sqrt{(l+m)(l-m+1)} |\psi_{l, m-2, s, \frac{m-2}{2}}\rangle + \alpha_m \sqrt{(l+m)(l-m+1)} |\psi_{l, m-2, s, \frac{m-2}{2}}\rangle + \beta_m \sqrt{(l+m)(l-m+1)} |\psi_{l, m-2, s, \frac{m-2}{2}}\rangle$

$\alpha_{m-1} \sqrt{(j+m)(j-m+1)} = \alpha_m \sqrt{(l+m-\frac{1}{2})(l-m+\frac{3}{2})}$
 $j = l + \frac{1}{2}$

$\alpha_{m-1} \sqrt{(l+m-\frac{1}{2})(l-m+\frac{3}{2})} = \alpha_m \sqrt{(l+m-\frac{1}{2})(l-m+\frac{3}{2})}$
 $j = m' \leq j$
 $m'_{max} = l + \frac{1}{2}$

$\frac{\alpha_m}{\sqrt{(l+m-\frac{1}{2})}} = \frac{\alpha_{m-1}}{\sqrt{(l+m-1)+\frac{1}{2}}} = \frac{\alpha_{m-1}}{\sqrt{(l+m-1)+\frac{1}{2}}} = \frac{\alpha_{m-1}}{\sqrt{(l+m-1)+\frac{1}{2}}} = \frac{\alpha_{m-1}}{\sqrt{(l+m-1)+\frac{1}{2}}} = \frac{\alpha_{m-1}}{\sqrt{(l+m-1)+\frac{1}{2}}}$

$\frac{\alpha_m}{\sqrt{(l+m-\frac{1}{2})}} = \frac{1}{\sqrt{2l+1}} \Rightarrow \alpha_m = \sqrt{\frac{l+m-\frac{1}{2}}{2l+1}}$

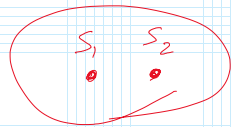
$$J_m = L_m + J_m = \downarrow$$

$$J = L + S \quad \left\{ \begin{array}{l} j = j_1 + j_2 \\ j = j_1 + j_2 - 1 \\ j = j_1 + j_2 - 2 \\ \dots \\ j = |j_1 - j_2| \end{array} \right.$$

$$J = J_1 + J_2$$

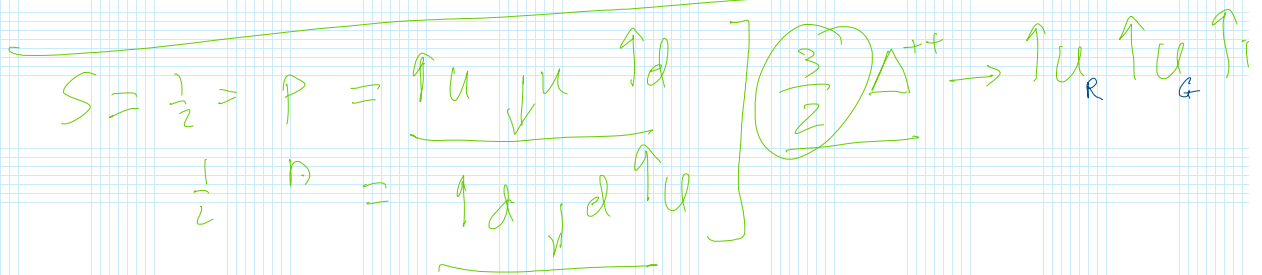
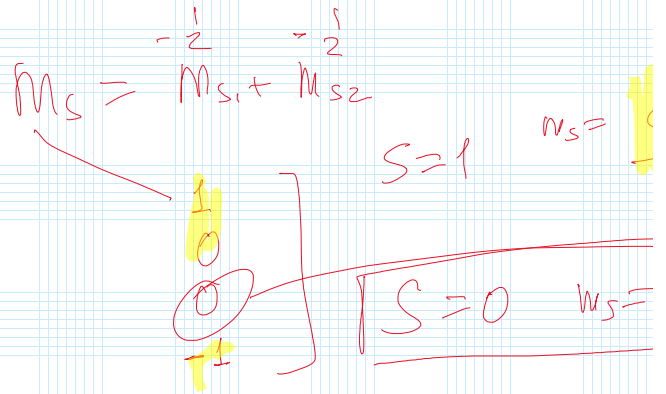
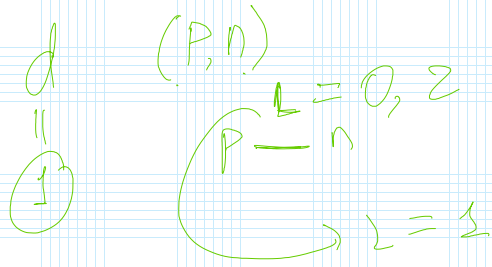
$$M_j = M_{j_1} + M_{j_2}$$

Example



$$S = s_1 + s_2$$

$S = 1 \leftarrow \begin{matrix} m_{s_1} \\ 0 \end{matrix} \right\}$ triplet state
 $S = 0 \rightarrow \begin{matrix} m_s \\ 0 \end{matrix} \right\}$ singlet



⇒ Recursion Formul.

$$J = J_1 + J_2$$

$$| \Phi \dots \rangle \quad | \psi_{j_1, m_{j_1}, j_2, m_{j_2}} \rangle$$

$$|^{-1} \psi_{m_j} \rangle \quad \begin{matrix} \uparrow J_{\pm} \\ | \psi_{m_j} \rangle \end{matrix} \quad \begin{matrix} \uparrow J_{\pm} \\ | \psi_{m_j} \rangle \end{matrix} \quad J_{\pm} | \psi_{m_j} \rangle = \sqrt{j(j+1) - m_j(m_j \pm 1)} | \psi_{m_j \pm 1} \rangle$$

$$\langle \psi_{j_1, m_1, j_2, m_2} | J_{\pm} | \psi_{m_j} \rangle = \sqrt{j(j+1) - m_j(m_j \pm 1)} \langle \psi_{j_1, m_1, j_2, m_2} | \psi_{m_j \pm 1} \rangle$$

$$\langle \psi_{j_1, m_1, j_2, m_2} | J_{1\pm} + J_{2\pm} | \psi_{m_j} \rangle$$

$$(J_{1\pm} + J_{2\pm}) | \psi_{j_1, m_1, j_2, m_2} \rangle = \sqrt{j_1(j_1+1) - m_1(m_1 \pm 1)} | \psi_{j_1, m_1 \pm 1, j_2, m_2} \rangle + \sqrt{j_2(j_2+1) - m_2(m_2 \pm 1)} | \psi_{j_1, m_1, j_2, m_2 \pm 1} \rangle$$

$$\begin{aligned} & \sqrt{j_1(j_1+1) - m_1(m_1 \pm 1)} \langle \psi_{j_1, m_1 \pm 1, j_2, m_2} | \psi_{m_j} \rangle + \sqrt{j_2(j_2+1) - m_2(m_2 \pm 1)} \langle \psi_{j_1, m_1, j_2, m_2 \pm 1} | \psi_{m_j} \rangle = \\ & = \sqrt{j(j+1) - m_j(m_j \pm 1)} \langle \psi_{j_1, m_1, j_2, m_2} | \psi_{m_j \pm 1} \rangle \end{aligned}$$

$$| \phi_{j, m_j} \rangle = \sum_{m_1, m_2} C_{j_1, m_1; j_2, m_2}^{j, m_j} | \psi_{j_1, m_1, j_2, m_2} \rangle$$

$$C_{j, m_j}^{j, m_j} = \langle \psi_{j, m_j} | \phi_{j, m_j} \rangle \quad | \psi \rangle = \sum \alpha$$

Clebsch - Gordan Coefficients

$$m_j = j$$

$$C_{j_1, j_1; j_2, j_2}^{j, j} = 1$$

$$M_1 = J_1 \quad J_2$$

$$J_1 = \frac{3}{2} \quad J_2 = 1$$