L15

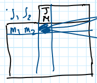
Wednesday, March 10, 2021 5:03 PM

$$C^{\frac{5}{2}\,\frac{3}{2}}_{1\,1\,\frac{3}{2}\,\frac{1}{2}}$$

$J \quad m$, $j_1 \quad j_2$

9

$$\| \quad \sqrt{\frac{3}{5}}$$



$\sqrt{N}$

$\sqrt{N}$

$$-K = -\sqrt{K}$$

$$C^{\frac{5}{2}\,\frac{1}{2}}_{1\,1\,\frac{3}{2}\,-\frac{1}{2}} = \sqrt{\frac{3}{10}} \qquad C^{\frac{5}{2}\,\frac{3}{2}}_{\underset{j_1 m_1}{1\,0}\,\underset{j_2 m_2}{\frac{3}{2}\,\frac{3}{2}}} = \sqrt{\frac{2}{5}}$$

$$C^{\frac{5}{2}\,-\frac{1}{2}}_{2\,-2,\,\frac{3}{2}\,\frac{3}{2}} = -\sqrt{\frac{6}{35}}$$

S
j_2

Two Particle System:

$m_1 \quad m_2$

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(r_1 - r_2)$$

$$r = r_1 - r_2, \qquad \vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$P_{rel} = \frac{m_2 P_1}{M_T} - \frac{m_1 P_2}{M_T} \quad \Bigg| \quad P_{CM} = \vec{P}_1 + \vec{P}_2$$

$$H = \frac{P_{CM}^2}{2M_T} + \frac{P_{rel}^2}{2\mu} + V(r)$$

$$\Psi(r, R_{CM}) = C e^{\frac{i\vec{P}_{CM}\vec{R}_{CM}}{\hbar}} \Psi_n(r_{rel})$$

$$E = \varepsilon + \frac{P_{CM}^2}{2M_{TOT}}$$

$$\left[\frac{P_{rel}^2}{2\mu} + V(r_{rel})\right]\Psi_n(r_{rel}) = \varepsilon\Psi_n(r_{rel})$$

$$\mu_{H^+} = 0.99945\ m_e \qquad E_n = -\frac{\alpha^3 m_e c^2}{2n^2}$$

$$e^2 = \alpha\hbar c$$

$$\mu_D = 0.99973\ m_e \qquad \frac{E_2 - E_1}{h} = \frac{\alpha^2 m_e c^2}{2}\left(1 - \frac{1}{4}\right)$$

– Considering mass of Nucleus

For Hydrogen

$\nu = \alpha^2 \mu_H c^2\ 3$

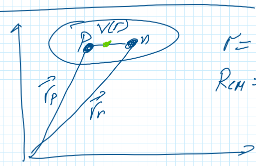
$\nu_H = \frac{\alpha^2 m_H c^2}{2}\frac{1}{4}$

For Deuterium $\quad \nu_D = \frac{\alpha^2 m_D c^2}{2}\frac{3}{4}$

$J = j_1 + j_2$	1	2
j_1+j_2	j_1	j_2
j_1+j_2-1		
- - -		
$\lvert j_1-j_2\rvert$		

$\Rightarrow$ Deuteron $\rightarrow$ p, n $\quad$ p, n

S	$s_p = \frac{1}{2}$ $\quad s_p = \frac{1}{2}$.
$\rightarrow$ 1	
$\rightarrow$ 0	



$r = \vec{r}_p - \vec{r}_n$

$R_{CM} = \frac{\vec{r}_p + \vec{r}_n}{2}$

$p = \frac{\vec{P}_p - \vec{P}_n}{2}$

$\boxed{P_{CM} = \frac{\hat{P}_p + \hat{P}_n}{2}}$

$$\hat{H} = \frac{\hat{P}_p^2}{2m} + \frac{\hat{P}_n^2}{2m} + V(\vec{r}_p - \vec{r}_n)$$

$$\hat{H}\,\Psi(r_p, s_p, r_n, s_n) = E\,\Psi(r_p, s_p, r_n, s_n)$$

$\Rightarrow$ introduce

$$\boxed{H_{CM} = \frac{\hat{P}_{CM}^2}{4m}}$$

$$\boxed{H_{int} = \frac{\hat{p}^2}{m} + V(r)}$$

$$(H_{CM} + H_{int})\,\Psi_d(r_p, s_p, r_n s_n) = E\,\Psi_D(r_p s_p, r_n s_n)$$

$$\Psi_d(r_p s_p, r_n, s_n) = e^{+i\frac{P_{CM} R_{CM}}{\hbar}}\,\Psi_d^{int}(r, s_p, s_n)$$

$$\boxed{H_{int}\,\Psi_d^{int}(r, s_p, s_n) = \underbrace{\left(E - \frac{P_{CM}^2}{4m}\right)}_{E_B}\Psi_D^{in}(r, s_p, s_n)}$$

Lab.

$$\boxed{H_{int}\,\Psi_D^{int}(r, s_p, s_n) = E_B\,\Psi_D^{in}(r, s_p, s_n)}$$

$$\Psi_D^{int}(r, s_p, s_n) = \boxed{R_l(r)\,Y_l^m(\theta, \varphi)\,S(s_p s_n)}$$

know

$l = ?$

— Experimentally $\quad J = 1 \qquad J = L + S$

$J=1$	L	S
1	0	1
1	~~1~~	~~1~~
1	2	1
1	~~3~~	1
1	~~1~~	~~0~~

J	s_1	s_2
2, 1, 0	1	1

J	s_1	s_2
s_1+s_2		
s_1+s_2-1		
- - -		
$\lvert s_1-s_2\rvert$		

— Deuteron has Positive Parity

$\vec{r} \rightarrow \vec{r}$

$$\boxed{\Psi_{Jn}(\vec{r}) = \Psi_n(\vec{r})}$$

$\vec{r} \rightarrow -\vec{r}$

$\theta \rightarrow \pi - \theta$

$\varphi \to \pi + \phi$ Inward Bound.

$Y_\ell^m(\pi-\theta, \pi+\phi) = (-1)^\ell Y_\ell^m(\theta,\varphi)$

$\ell = \text{even}$ $\quad \ell = 0, \quad \ell = 2, \quad S = 1 \quad C^{jm}_{\ell m_\ell, s m_s}$

$$\Psi^{jm\;11}_{in}(r, s_p, s_n) = C^{11}_{00;11} R_0(r) Y_0^0(\theta,\varphi)|1,1\rangle + C^{11}_{21;10} R_2(r) Y_2^1(\theta,\varphi)|1,0\rangle + C^{11}_{20;11} R_2(r) Y_2^0(\theta,\varphi)|1$$
$$C^{11}_{22;1-1} R_2(r) Y_2^2(\theta,\varphi)|1-1\rangle +$$

$\Psi^{10}_{in}(r, s_p, s_n) =$

$\Psi^{1-1}_{in}(r, s_p s_n) =$