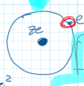
L6

Monday, February 8, 2021 4:54 PM

Hydrogenlike Atoms



$V(r) = -\frac{Ze^2}{r}$ $e^2 = \alpha \hbar c$, $\alpha = \frac{1}{137.030}$

F. Const, F. Const, F. const

$\hat{H} = \frac{p^2}{2m} + V(r) = \frac{p^2}{2m} - \frac{Ze^2}{r}$

$F = k\frac{q_1 q_2}{r^2}$

$F = \frac{q_1 q_2}{r^2}$

$r \to \infty$ $V(r)\Big|_{r\to\infty} \longrightarrow 0$

$\boxed{\hat{H}\Psi_{Eem}(r,\theta,\varphi) = E\,\Psi_{Eem}(r,\theta,\varphi)}$

$\boxed{E < 0}$

1. Spherically Symmetric.
2. $\Psi_{E\ell m}(r,\theta,\varphi) = R_{E\ell}(r)\, Y_\ell^m(\theta,\varphi)$
3. $E = ?$, $R_{E\ell}(r) = ?$

$E = ?$ Method of Commuting Operators $\hat{O}$

$[\hat{O}\hat{H}] = 0$

$\hat{O} = \hat{L}^2, \; L_z$

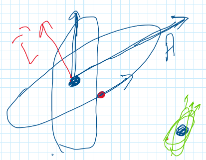
$\boxed{\begin{aligned}[\hat{L}^2, \hat{H}] &= 0 \\ [L_z, \hat{H}] &= 0\end{aligned}}$

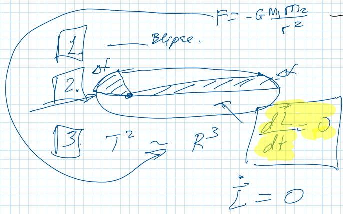
$[\hat{A}\,\hat{H}] = 0$

$\boxed{\hat{A}\,\Psi_{E\ell}(\vec{r}) = a\Psi_{E\ell}(r_m)}$

$\hat{A} \sim \hat{H}$ solve for E

⇒ Planetary System





$F = -G\frac{M_1 m_2}{r^2} \longrightarrow$ $\boxed{\begin{aligned}V_G(r) &= -\frac{GMm}{r} \\ V_c(r) &= -\frac{Ze^2}{r}\end{aligned}}$

$\dot{\vec{L}} = 0$

$V(r) = -\frac{K}{r}$

Keplerian Problem

$\frac{d}{dt}\vec{A} = 0$ $\vec{A} = ?$ $\Big|$ $\frac{d\vec{L}}{dt} = 0$

$[\hat{A}_i\,\hat{H}] = 0$

⇒ Kepler 1571 – 1630 – 3 laws 1609 $t^2 \sim R^3$

Classical Physics Keplerian

$L^{cl} \longrightarrow \hbar\hat{L}$

Laplace-Runge-Lenz Vector

$$\boxed{\vec{A} = \vec{L}\times\vec{P} + Km\frac{\vec{r}}{r}}$$

$V(r) = -\frac{K}{r}$

$$\boxed{\vec{L}\cdot\vec{A} = 0} = \vec{L}\cdot(\vec{L}\times\vec{P}) + Km\frac{\vec{L}\cdot\vec{r}}{r}$$

$\hat{L}\times(L\times p) = $

$$\boxed{\vec{L} = (r\times p)}$$

$$\boxed{\frac{d\vec{A}}{dt}} = \dot{\vec{A}} = \cancel{\dot{\vec{L}}\times P} + \vec{L}\times\dot{\vec{P}} - Km\frac{d}{dt}\frac{\vec{r}}{r}$$

$$\frac{d\vec{A}}{dt} = {}^{\mathrm{I}}\,\boxed{\vec{L}\times\frac{d\vec{P}}{dt}} + \boxed{Km\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)}$$ $\frac{d}{dt}\left(\frac{\vec{r}}{r}\right) = \frac{\dot{\vec{r}}}{r} - \vec{r}\frac{\dot{r}}{r^2}$

$$\frac{d\vec{P}}{dt} = \vec{F} = -\vec{\nabla}V(r) = -\vec{\nabla}\left(-\frac{K}{r}\right) = K\vec{\nabla}\frac{1}{r} = -K\frac{\vec{r}}{r^3}$$

$$\vec{L} = r\times p = \vec{r}\times m\dot{\vec{r}} = m(\vec{r}\times\dot{\vec{r}})$$

$$(A\times B)\times C = \vec{B}(A\cdot C) - \vec{C}(A\cdot B)$$

$$\mathrm{I} = m(\vec{r}\times\dot{\vec{r}})\times(-K)\frac{\vec{r}}{r^3} = -\frac{mK}{r^3}(\vec{r}\times\dot{\vec{r}})\times\vec{r} = -\frac{mK}{r^3}\left(\dot{\vec{r}}(r^2) - \vec{r}(\vec{r}\cdot\dot{\vec{r}})\right)$$

$+$

$$\mathrm{II} = \frac{Km\dot{\vec{r}}}{r} - Km\frac{\vec{r}\cdot\dot{r}}{r^2}$$

$$\frac{d}{dt}\sqrt{x^2+y^2+z^2} =$$

$$\frac{dA}{dt} = -\cancel{mK\frac{\dot{\vec{r}}}{r}} + mK\frac{\vec{r}(\vec{r}\cdot\dot{\vec{r}})}{r^3} + \cancel{\frac{Km\dot{\vec{r}}}{r}} - Km\frac{\vec{r}\,\dot{r}}{r^2}$$

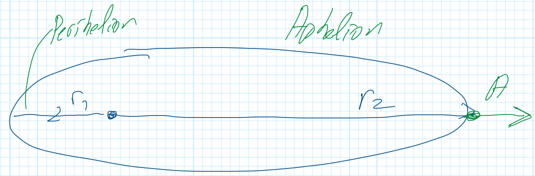
$$|r| = \sqrt{x^2+y^2+z^2}$$

$$\boxed{\vec r \cdot \dot{\vec r} = r|\dot r|} = r_x \frac{dr_x}{dt} + r_y \frac{dr_y}{dt} + r_z \frac{dr_z}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt} = \frac12 \frac{dx^2}{dt} + \frac12 \frac{dy^2}{dt} + \frac12 \frac{dz^2}{dt}$$

$$= \frac12 \frac{d(x^2+y^2+z^2)}{dt} = \frac12 \frac{dr^2}{dt} = r\frac{dr}{dt}$$

$$\frac{d\vec A}{dt} = \frac{mk\, \vec r \cdot \dot{\vec r}\, \vec r}{r^3} - \frac{mk\, \dot{\vec r}}{r^2} = 0$$

Direction



Quantum Mechanics

1926 Wolfgang Pauli

$$\hat A_i^{QM} = ? \qquad [\hat H, \hat A_i^{QM}] = 0$$

$$L_i \to \hbar L_i$$ integers

$$\hat A_i = (\hbar \hat L \times \hat p)_i + \frac{km \hat r_i}{r}$$

$$\hat A^\dagger = (\hbar \hat p \times \hat L)_i + \frac{km \hat r_i}{r}$$

$$\boxed{\hat A \neq \hat A^\dagger}$$

$$(A \cdot B)^\dagger = B^\dagger A^\dagger$$

$$A_i^\dagger \neq A_i$$

$$\hat A_i = \hbar \sum \varepsilon_{ijk} \hat L_j \hat P_k + \frac{km \hat r_i}{r}$$

$$\boxed{[\hat P_k \hat L_j] = i \sum_m \varepsilon_{kjm} \hat P_m} = i \sum_m \varepsilon_{kjm} \hat P_m$$

$$A_i^\dagger = \hbar \sum \varepsilon_{ijk} \hat P_k^\dagger \hat L_j^\dagger + \frac{km \hat r_i^\dagger}{r} = \hbar \sum \varepsilon_{ijk} \hat P_k \hat L_j + \frac{km \hat r_i}{r}$$

$$P_k L_j = \hat L_j \hat P_k + i \sum_m \varepsilon_{kjm} P_m$$

$$A_i^\dagger = \hbar \sum \varepsilon_{ijk} L_j P_k + \hbar \sum_k \varepsilon_{ijk}\, i \sum_m \varepsilon_{kjm} P_m + \frac{km r_i}{r}$$

$$\boxed{\hat A_i^{QM} = \frac12 (\hbar L \times P)_i - \frac12 (P \times \hbar L)_i + km \frac{r_i}{r}}$$

$$\hat L^\dagger = \hat L, \quad \hat r^\dagger = \hat r, \quad \hat p^\dagger = p$$

$$A_i^{QM} = A_i^{QM\dagger}$$

$$\boxed{\hat A_i^{QM\dagger} = \frac12 (P \times \hbar L)_i - \frac12 (\hbar L \times P)_i + km \frac{r_i}{r}}$$

$$\boxed{[P_i L_j] = i \sum \varepsilon_{ijk} P_k}$$

$$A_i^{QM\dagger} = \frac{\hbar}{2} \sum_i \varepsilon_{ijk} P_j L_k - \frac{\hbar}{2} \sum \varepsilon_{ijk} L_j P_k + km \frac{r_i}{r}$$

$\Longrightarrow$ Classical Limit

$$\boxed{A_i} = \underbrace{\frac{1}{2}(L\times P) \overbrace{- \frac{1}{2}(P\times L)}^{-\frac{1}{2}(L\times P)}}_{\boxed{(L\times P)}} + \frac{K m r_i}{r}$$

$\Longrightarrow$ Hydrogen Like Atoms $\quad K = Ze^2$

$$A_i = \frac{1}{2}(\hbar L \times P)_i - \frac{1}{2}(P\times \hbar L)_i + Ze^2 m \frac{r_i}{r}$$

$[H A_i] = 0 \Longrightarrow [H A^2] = 0$ HW

$[L_i, A_j] = i \sum \varepsilon_{ijk} A_k$

$[A_i\ A_j] = -2im\hbar^2 H \sum_k \varepsilon_{ijk} L_k$

$[L^2, A_i] = 0$

$[L^2, A^2] = 0$

$\Longrightarrow$ Use $A_i \longrightarrow E$ – Hydrogen Like Atom.